问题与反馈

2015.5.15

Exercise 4.2.1.4. Change the greedy strategy of the algorithm GMS to an arbitrary choice, i.e., without sorting $p_1, p_2, \ldots, p_n$ (removing Step 1 of GMS), assign the jobs in the on-line manner[2] as described in Step 2 and 3 of GMS. Prove that this simple algorithm, called GRAHAM'S ALGORITHM, is a 2-approximation algorithm for MS, too. □

Exercise 4.2.1.5. Find, for every integer $m \geq 2$, an input instance I_m of MS such that $R_{\text{GMS}}(I) = \frac{cost(\text{GMS}(I))}{Opt_{\text{MS}}(I)}$ is as large as possible. □

Exercise 4.2.3.3. Prove that the functions *dist*, $dist_k$, and *distance* (defined in Example 4.2.3.2) are distance functions for TSP according to $L_\triangle$. □

$$dist(G,c) = \max\left\{0, \max\left\{\frac{c(\{u,v\})}{c(\{u,p\}) + c(\{p,v\})} - 1 \,\middle|\, u,v,p \in V(G),\right.\right.$$
$$\left.\left. u \neq v, u \neq p, v \neq p\right\}\right\},$$
$$dist_k(G,c) = \max\left\{0, \max\left\{\frac{c(\{u,v\})}{\sum_{i=1}^{m} c(\{p_i,p_{i+1}\})} - 1 \,\middle|\, u,v \in V(G) \text{ and}\right.\right.$$
$$u = p_1, p_2, \ldots, p_m = v \text{ is a simple path between } u \text{ and } v$$
$$\left.\left.\text{of length at most } k \text{ (i.e., } m+1 \leq k)\right\}\right\}$$

for every integer $k \geq 2$, and

$$distance(G,c) = \max\{\, dist_k(G,c) \mid 2 \leq k \leq |V(G)| - 1\}.$$

Definition 4.2.3.1. *Let* $U = (\Sigma_I, \Sigma_O, L, L_I, \mathcal{M}, cost, goal)$ *and* $\overline{U} = (\Sigma_I, \Sigma_O, L, L, \mathcal{M}, cost, goal)$ *be two optimization problems with* $L_I \subset L$. *A* **distance function for** $\overline{U}$ **according to** L_I *is any function* $h_L : L \to \mathbb{R}^{\geq 0}$ *satisfying the properties*

(i) $h_L(x) = 0$ *for every* $x \in L_I$, *and*
(ii) h *is polynomial-time computable.*

Let h *be a distance function for* $\overline{U}$ *according to* L_I. *We define, for any* $r \in \mathbb{R}^+$,

$$\boldsymbol{Ball_{r,h}(L_I)} = \{w \in L \mid h(w) \leq r\}.$$ [6]

Let A *be a consistent algorithm for* $\overline{U}$, *and let* A *be an* ε*-approximation algorithm for* U *for some* $\varepsilon \in \mathbb{R}^{>1}$. *Let* p *be a positive real. We say that* A *is* **p-stable according to h** *if, for every real* $0 < r \leq p$, *there exists a* $\delta_{r,\varepsilon} \in \mathbb{R}^{>1}$ *such that* A *is a* $\delta_{r,\varepsilon}$*-approximation algorithm for* $U_r = (\Sigma_I, \Sigma_O, L, Ball_{r,h}(L_I), \mathcal{M}, cost, goal)$.

A is **stable according to h** *if A is p-stable according to h for every* $p \in \mathbb{R}^+$. *We say that A is* **unstable according to h** *if A is not p-stable for any* $p \in \mathbb{R}^+$.

For every positive integer r, *and every function* $f_r : \mathbb{N} \to \mathbb{R}^{>1}$ *we say that* A *is* $\boldsymbol{(r, f_r(n))}$**-quasistable according to h** *if* A *is an* $f_r(n)$*-approximation algorithm for* $U_r = (\Sigma_I, \Sigma_O, L, Ball_{r,h}(L_I), \mathcal{M}, cost, goal)$.

Exercise 4.2.3.4. Let $U = (\Sigma_I, \Sigma_O, L, L_I, \mathcal{M}, cost, goal)$ and $\overline{U} = (\Sigma_I, \Sigma_O, L, L, \mathcal{M}, cost, goal) \in \text{NPO}$. Let h_{index} be defined as follows:

(i) $h_{index}(w) = 0$ for every $w \in L_I$, and
(ii) $h_{index}(u)$ is equal to the order of u according to the canonical order of words in Σ_I^*.

Prove that

a) h_{index} is a distance function of $\overline{U}$ according to L_I.
b) For every δ-approximation algorithm A for U, if A is consistent for $\overline{U}$, then A is stable according to h_{index}. □

An algorithm A is **consistent** *for U if, for every $x \in L_I$, the output $A(x) \in \mathcal{M}(x)$. We say that an algorithm B* **solves** *the optimization problem U if*

Exercise 4.2.3.3 shows that it is not interesting to consider a distance function h with the property

$$|Ball_{r,h}(L_I)| - |Ball_{q,h}(L_I)| \text{ is finite}$$

for every $r > q$. The distance function h' investigated later has the following additional property (called the **property of infinite jumps**):

"If $Ball_{q,h'}(L_I) \subset Ball_{r,h'}(L_I)$ for some $q < r$,
then $|Ball_{r,h'}(L_I)| - |Ball_{q,h'}(L_I)|$ is infinite."

Exercise 4.2.3.5. Define two optimization problems $U = (\Sigma_I, \Sigma_O, L, L_I, \mathcal{M}, cost, goal)$ and $\overline{U} = (\Sigma_I, \Sigma_O, L, L, \mathcal{M}, cost, goal)$ from NPO with infinite $|L| - |L_I|$, and a distance function h for $\overline{U}$ according to L_I such that:

(i) h has the property of infinite jumps, and
(ii) for every δ-approximation algorithm A for U, if A is consistent for $\overline{U}$, then A is stable according to h. □

Lemma 4.3.5.9. CHRISTOFIDES ALGORITHM *is stable according to distance.*