计算机问题求解 – 论题2–5

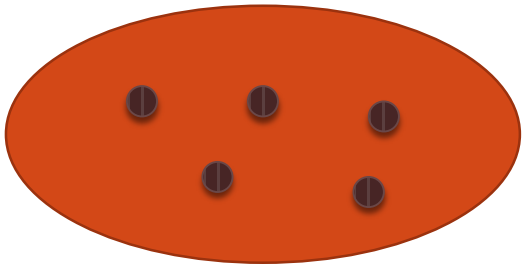
- 离散概率基础

  课程研讨

  - CS第5章第1-4节

问题1： probability

- 你理解这些概念了吗？
  - Sample space
  - Element
  - Event
  - Probability weight
  - Probability



- 你能基于这些概念解释probability distribution function的三个条件吗？

1. $P(A) \geq 0$ for any $A \subseteq S$.
2. $P(S) = 1$.
3. $P(A \cup B) = P(A) + P(B)$ for any two disjoint events A and B.

问题1： probability (续)

- 在这些例子中，sample space、element、event分别是什么？
  - The probability of getting at least 1 head in 5 flips of a coin.
  - The probability of getting a total of 6 or 7 on the 2 dice.
  - The probability that all 3 keys hash to different locations (among 20).

- 你能给出它们的答案吗？

问题1： probability (续)

- 你理解uniform probability distribution了吗？

Theorem 5.2 *Suppose P is the uniform probability measure defined on a sample space S. Then for any event E,*

$$P(E) = |E|/|S|,$$

the size of E divided by the size of S.

- 现在你能给出之前几题的答案了吗？
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- uniform probability distribution为计算带来了怎样的便利？

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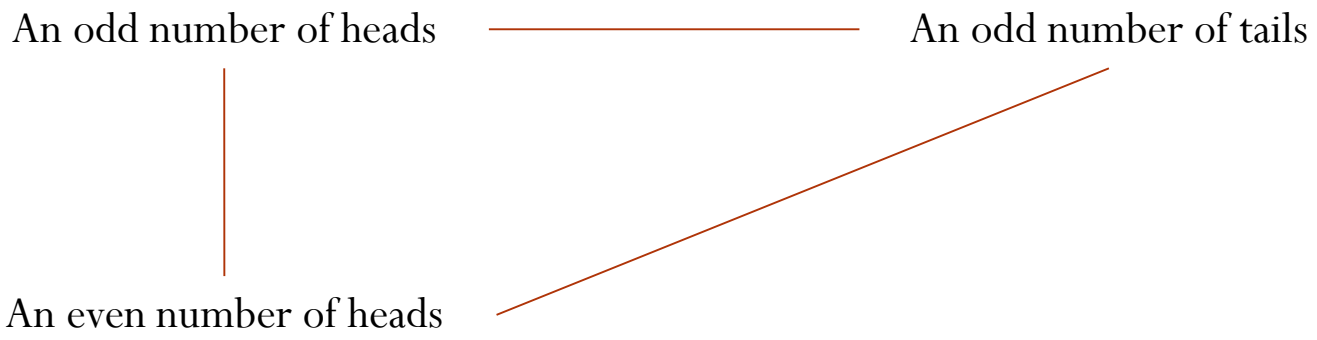
  probability → counting

问题1： probability (续)

- What is the probability of an odd number of heads in three tosses of a coin?（假设是uniform probability distribution）
  - 如何利用这个三角形快速求解？



- 如果不是uniform probability distribution，怎么办？

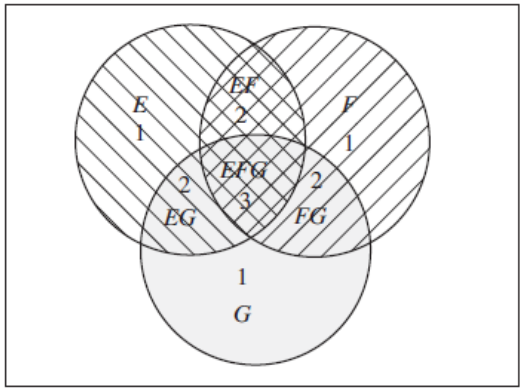
问题1： probability (续)

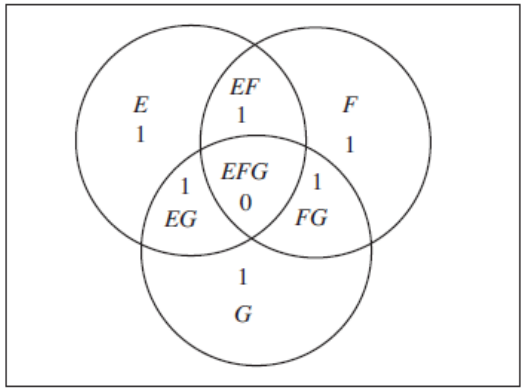
- Which is more likely, or are both equally likely?
  - Drawing an ace and a king when you draw two cards from among the 13 spades, or drawing an ace and a king when you draw two cards from an ordinary deck of 52 playing cards?
  - Drawing an ace and a king of the same suit when you draw two cards from a deck, or drawing an ace and a king when you draw two cards from among the 13 spaces?

问题2：the principle of inclusion and exclusion

- 你理解这两个图的含义了吗？

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(F \cap G) + P(E \cap F \cap G)$$





- 你读懂这个公式了吗？

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{\substack{i_1, i_2, \ldots, i_k: \\ 1 \le i_1 < i_2 < \cdots < i_k \le n}} P\left(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k}\right)$$

问题2：the principle of inclusion and exclusion (续)

- How many functions from an *m*-element set *M* to an *n*-element set *N* map nothing to at least one element of *N*?
  - Sample space?
  - Element?
  - Event?

$$\left|\bigcup_{i=1}^{n} E_i\right| = \sum_{k=1}^{n} (-1)^{k+1} \sum_{\substack{i_1, i_2, \ldots, i_k: \\ 1 \le i_1 < i_2 < \cdots < i_k \le n}} |E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k}|$$

E_i是什么？

在这里如何计算？

问题2： the principle of inclusion and exclusion (续)

- How many functions from an *m*-element set *M* to an *n*-element set *N* map nothing to at least one element of *N*?
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不映射到i的函数集合

$$\binom{m}{k}(m-k)^n$$

问题2： the principle of inclusion and exclusion (续)

- In how many ways may you distribute k identical apples to n children so that no child gets more than m?

问题2：the principle of inclusion and exclusion (续)

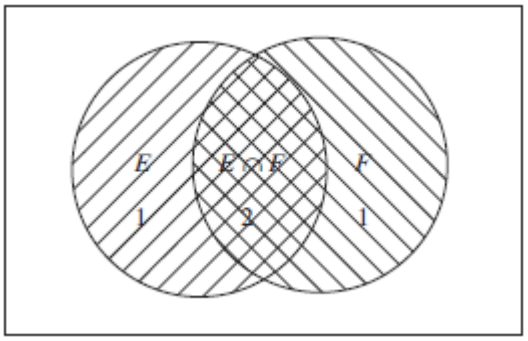
- In how many ways may you distribute k identical apples to n children so that no child gets more than m?

$$\binom{k+(n-1)}{n-1} - \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} \binom{k-(m+1)i+(n-1)}{n-1}$$

问题3： conditional probability

- 你能结合Venn图解释条件概率的定义吗？

$$P(E|F) = \frac{P(E \cap F)}{P(F)}.$$



- 你能结合图解释独立性吗？

$$P(E|F) = P(E)$$

- 你能自己推导出这两个定理吗？

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}.$$

Theorem 5.4 *Suppose E and F are events in a sample space. Then E is independent of F if and only if $P(E \cap F) = P(E)P(F)$.*

问题3： conditional probability (续)

$$P(x_i = a_i | x_1 = a_1, \ldots, x_{i-1} = a_{i-1}) = P(x_i = a_i)$$

- 你理解independent trials process了吗？

Exercise 5.3-7 Suppose we draw a card from a standard deck of 52 cards, discard it (i.e. we do not replace it), draw another card and continue for a total of ten draws. Is this an independent trials process?

- 为什么这不是一个independent trials process？
- 为这个过程绘制tree diagram，并计算：第*i*张抽到梅花A的概率是多少？
- 如果是independent trails process，其tree diagram有什么特征？

问题3： conditional probability (续)

- A nickel, two dimes, and two quarters are in a cup. We draw three coins, one at a time, without replacement.
  - Draw the probability tree which represents the process.
  - Use the tree to determine the probability of getting a nickel on the last draw.
  - Use the tree to determine the probability that the first coin is a quarter, given that the last coin is a quarter.

问题4：random variables

- 你理解这些概念了吗？能自己举个例子吗？
  - Random variable
  - Expected value

$$E(X+Y) = E(X) + E(Y)$$

- 你能直观解释它们为什么相等吗？

$$E(X) = \sum_{i=1}^{k} x_i P(X = x_i)$$

$$E(X) = \sum_{s:s \in S} X(s) P(s)$$

问题4：random variables (续)

- How many sixes do we expect to see on top if we roll 24 dice?

- What is the expected number of times we need to roll two dice until we get a 7?

问题4：random variables (续)

- A student is taking a true-false test and guessing when he doesn't know the answer. We are going to compute a score by subtracting a percentage of the number of incorrect answers from the number of correct answers. That is, for some number y, the student's corrected score will be

  (number of corrected answers) – y(number of incorrect answers)

  When we convert this "corrected score" to a percentage score, we want its expected value to be the percentage of the material being tested that the student knows. How can we do this?