- 作业讲解
  - JH第3章练习3.3.2.7、3.3.2.8、3.3.2.9

JH第3章练习3.3.2.9

Exercise 3.3.2.9. Let $((X, \mathcal{F}), k)$, $\mathcal{F} \subseteq Pot(X)$, be an instance of the decision problem $Lang_{SC}$.[10] Let, for every $x \in X$, $num_{\mathcal{F}}(x)$ be the number of sets in $\mathcal{F}$ that contain x. Define

$$Pat((X, \mathcal{F}), k) = \max\{k, \max\{num_{\mathcal{F}}(x) \mid x \in X\}\}$$

that is a parameterization of $Lang_{SC}$. Find a Pat-parameterized polynomial-time algorithm for $Lang_{SC}$. □

- 分治法
  - 每个x所属的$num_F(x)$个集合中必选至少一个
  - 最多选k个集合
  - 因此，总共不超过Pat^{Pat}种尝试

- 教材讨论
  - JH第3章第6节

问题1：local search的基本概念

- 你能解释这些术语的含义吗？
  基于此，你能解释local search的基本思想吗？
  - feasible solution
  - transformation
  - neighborhood
  - local optimum

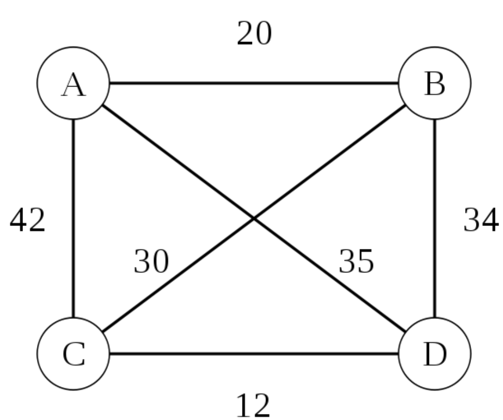
LSS(*Neigh*)-Local Search Scheme according to a neighborhood *Neigh*

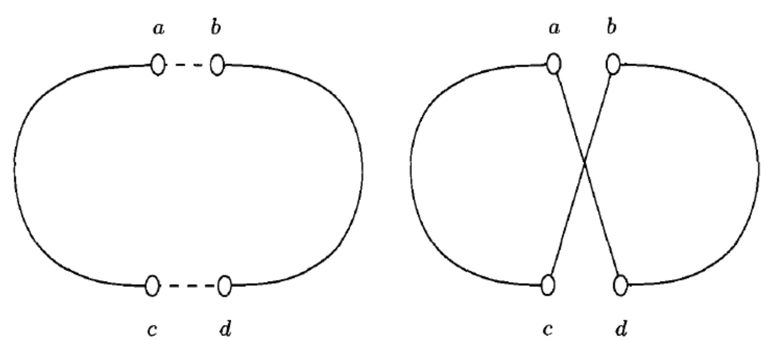
Input: An input instance x of an optimization problem U.
Step 1: Find a feasible solution $\alpha \in \mathcal{M}(x)$.
Step 2: **while** $\alpha \notin LocOPT_U(x, Neigh_x)$ **do**
  begin find a $\beta \in Neigh_x(\alpha)$ such that
    $cost(\beta) < cost(\alpha)$ if U is a minimization problem and
    $cost(\beta) > cost(\alpha)$ if U is a maximization problem; $\alpha := \beta$
  end
Output: **output**(α).

- 你能证明LSS的total correctness吗？
- 决定LSS能否并尽快找到全局最优解的因素有哪些？
  - α
  - Neigh
  - β

问题2：hill climbing

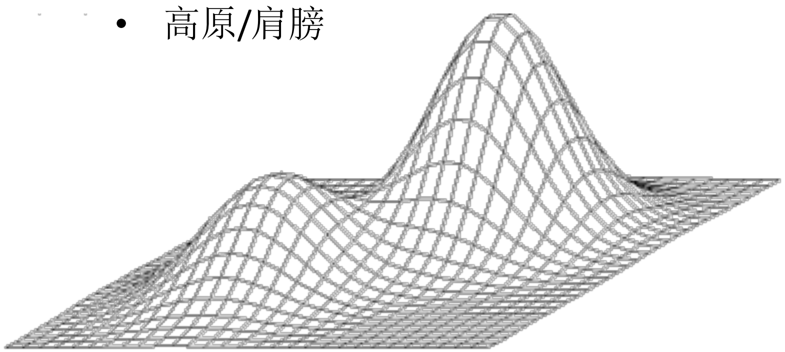
- In computer science, hill climbing is a mathematical optimization technique which belongs to the family of local search. It is an iterative algorithm that starts with an arbitrary solution to a problem, then attempts to find a better solution by incrementally changing a single element of the solution. If the change produces a better solution, an incremental change is made to the new solution, repeating until no further improvements can be found.
  - 这里的α、Neigh、β分别是怎么取的？
  - 你能以TSP为例，给出一个具体的算法吗？

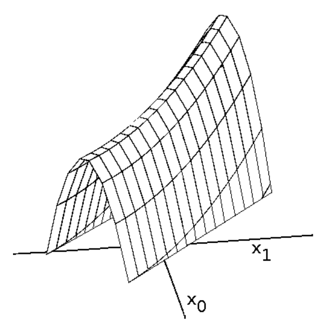


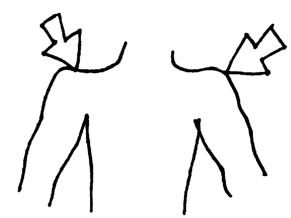


问题2： hill climbing (续)

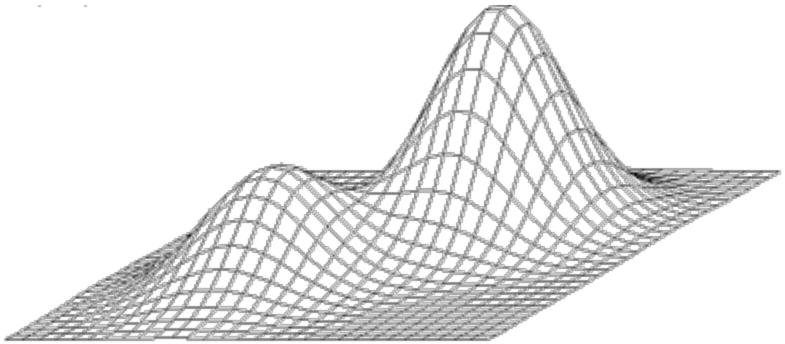
- In computer science, hill climbing is a mathematical optimization technique which belongs to the family of local search. It is an iterative algorithm that starts with an arbitrary solution to a problem, then attempts to find a better solution by incrementally changing a single element of the solution. If the change produces a better solution, an incremental change is made to the new solution, repeating until no further improvements can be found.
  - 你认为hill climbing存在哪些问题？
    - 局部最优
    - 缓升（如：之字形爬升非轴向的山脊）
    - 高原/肩膀





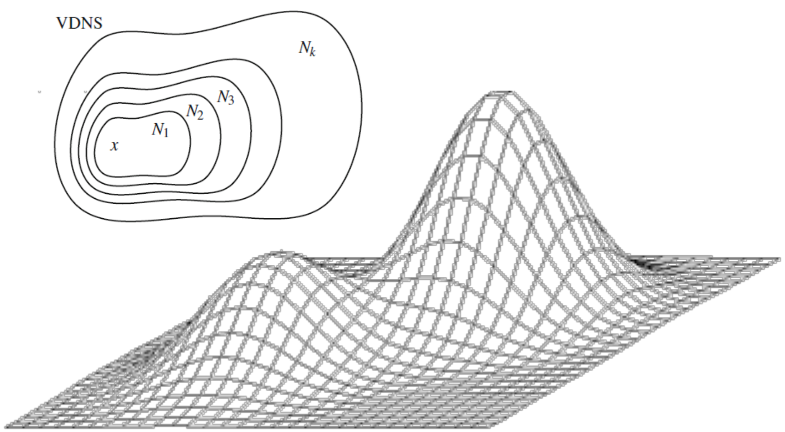


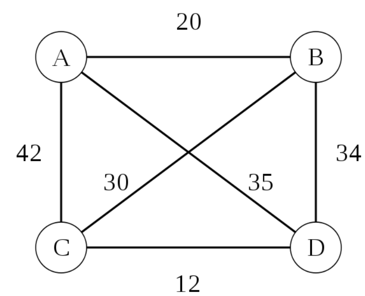
- 你能想到哪些策略来缓解这些问题？
  - α
  - Neigh
  - β

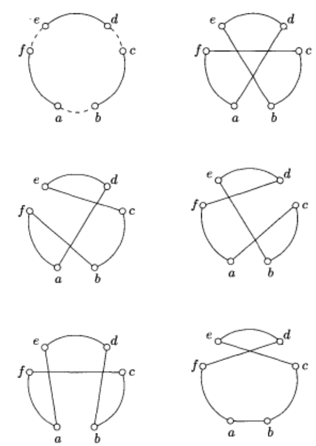


问题3：very large-scale neighborhood search

- A very large-scale neighborhood search is a local search algorithm which makes use of a neighborhood definition, which is large and possibly exponentially sized.
  - 和hill climbing相比，它改变了α、Neigh、β中的哪一个？这样做有什么好处？
  - 你能以TSP为例，给出一个具体的算法吗？







  - 你认为朴素的very large-scale neighborhood search存在什么问题？
  - 你能想出折中的策略来应对这个问题吗？

问题3：very large-scale neighborhood search (续)

- Variable-depth search methods are techniques that search the *k*-exchange neighborhood partially, hence reducing the time used to search the neighborhood.
  - KL(Neigh)是如何实现“partially”的？ 你能简述它的思想吗？
  - 你能以TSP为例，给出一个具体的算法吗？
  - 你还能想到其它方法来实现“partially”吗？

KL($Neigh$) Kernighan-Lin Variable-Depth Search Algorithm with respect to the neighborhood $Neigh$

Input: An input instance I of an optimization problem U.

Step 1: Generate a feasible solution $\alpha = (p_1, p_2, \ldots, p_n) \in \mathcal{M}(I)$ where $(p_1, p_2, \ldots, p_n)$ is such a parametric representation of α that the local transformation defining $Neigh$ can be viewed as an exchange of a few of these parameters.

Step 2: $IMPROVEMENT := TRUE$;
$EXCHANGE := \{1, 2, \ldots, n\}$; $J := 0$; $\alpha_J := \alpha$;
while $IMPROVEMENT = TRUE$ **do begin**
  while $EXCHANGE \neq \emptyset$ **do**
    begin $J := J + 1$;
      $\alpha_J :=$ a solution from $Neigh(\alpha_{J-1})$ such that $gain(\alpha_{J-1}, \alpha_J)$ is the maximum of
      $\{gain(\alpha_{J-1}, \delta) \mid \delta \in Neigh(\alpha_{J-1}) - \{\alpha_{J-1}\}$ and δ differs from α_{J-1} in the parameters of $EXCHANGE$ only$\}$;
      $EXCHANGE := EXCHANGE - \{$the parameters in which α_J and α_{J-1} differ$\}$
  end;
  Compute $gain(\alpha, \alpha_i)$ for $i = 1, \ldots, J$;
  Compute $l \in \{1, \ldots, J\}$ such that

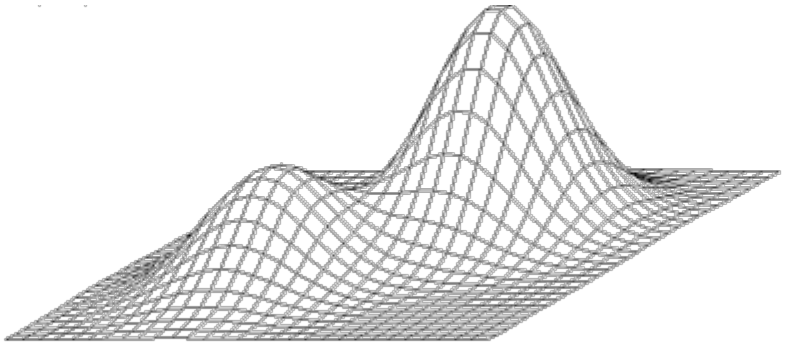
$$gain(\alpha, \alpha_l) = \max\{gain(\alpha, \alpha_i) \mid i \in \{1, 2, \ldots, J\}\};$$

  if $gain(\alpha, \alpha_l) > 0$ **then**
    begin $\alpha := \alpha_l$;
      $EXCHANGE := \{1, 2, \ldots, n\}$
    end
  else $IMPROVEMENT := FALSE$
end

Step 3: **output**(α).

问题4：Multi-start methods

- Re-start the procedure from a new solution once a region has been explored.
  - 和hill climbing相比，它改变了α、Neigh、β中的哪一个？这样做有什么好处？
  - 你认为应该如何选择new solution？
  - 你能以TSP为例，给出一个具体的算法吗？



问题4：Multi-start methods (续)

- Greedy Randomized Adaptive Search Procedure (GRASP)
  - The GRASP metaheuristic is a multi-start or iterative process, in which each iteration consists of two phases: construction and local search. The construction phase builds a feasible solution, whose neighborhood is investigated until a local minimum is found during the local search phase. The best overall solution is kept as the result.

```
procedure GRASP(Max_Iterations,Seed)
    Read_Input();
    for k = 1,...,Max_Iterations do
        Solution ← Greedy_Randomized_Construction(Seed);
        Solution ← Local_Search(Solution);
        Update_Solution(Solution,Best_Solution);
    end;
    return Best_Solution;
end GRASP.
```

FIGURE 1. Pseudo-code of the GRASP metaheuristic.

```
procedure Greedy_Randomized_Construction(Seed)
    Solution ← ∅;
    Evaluate the incremental costs of the candidate elements;
    while Solution is not a complete solution do
        Build the restricted candidate list (RCL);
        Select an element s from the RCL at random;
        Solution ← Solution ∪ {s};
        Reevaluate the incremental costs;
    end;
    return Solution;
end Greedy_Randomized_Construction.
```

FIGURE 2. Pseudo-code of the construction phase.

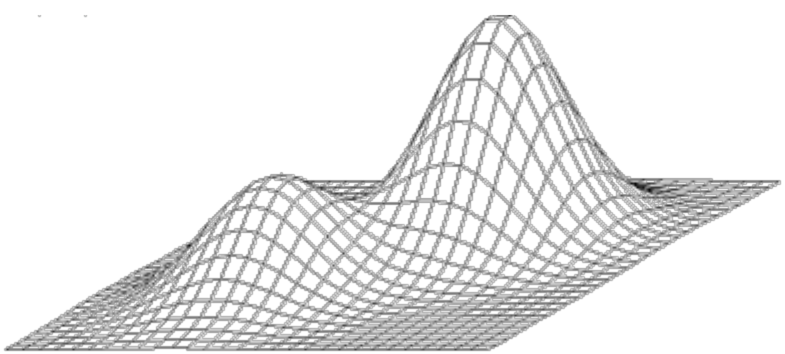
Greedy、Randomized、Adaptive 分别是如何体现的？

```
procedure Local_Search(Solution)
    while Solution is not locally optimal do
        Find s' ∈ N(Solution) with f(s') < f(Solution);
        Solution ← s';
    end;
    return Solution;
end Local_Search.
```

FIGURE 3. Pseudo-code of the local search phase.

问题5：Stochastic hill climbing

- Stochastic hill climbing chooses at random from among the uphill moves.
  - 和hill climbing相比，它改变了α、Neigh、β中的哪一个？这样做有什么好处？
  - 你认为应该如何计算每一种移动的概率？
    - The probability of selection can vary with the steepness of the uphill move.
  - 你能以TSP为例，给出一个具体的算法吗？



- 我们似乎已经讨论了对hill climbing的所有可能的改进策略
  - α：multi-start methods (GRASP)
  - Neigh：very large-scale neighborhood search (variable-depth search)
  - β：stochastic hill climbing
- 你还能想到别的方法吗？
  - 提示：计算机除了“算”以外，还能做什么？

问题6：Tabu search

- Tabu search enhances the performance of local searches by using memory structures that describe the visited solutions or user-provided sets of rules.
  - Short-term: The list of solutions recently considered. If a potential solution appears on the tabu list, it cannot be revisited until it reaches an expiration point.
  - Intermediate-term: Intensification rules intended to bias the search towards promising areas of the search space.
  - Long-term: Diversification rules that drive the search into new regions (i.e. regarding resets when the search becomes stuck in a plateau or a suboptimal dead-end).

- 你能以TSP为例，给出一个具体的算法吗？

问题7：local search的性能

LSS(*Neigh*)-Local Search Scheme according to a neighborhood *Neigh*

Input: An input instance x of an optimization problem U.
Step 1: Find a feasible solution $\alpha \in \mathcal{M}(x)$.
Step 2: **while** $\alpha \notin LocOPT_U(x, Neigh_x)$ **do**
  begin find a $\beta \in Neigh_x(\alpha)$ such that
    $cost(\beta) < cost(\alpha)$ if U is a minimization problem and
    $cost(\beta) > cost(\alpha)$ if U is a maximization problem; $\alpha := \beta$
  end
Output: **output**(α).

- local search的运算时间受哪些因素的影响？

- 什么是exact polynomial-time searchable neighborhood？为什么cost-bounded integer-valued TSP没有这一性质？

问题8：应用

- 你能综合运用我们讨论的这些策略，分别为下列问题设计一种local search算法吗？
  - longest simple path
  - MAX-SAT
  - MAX-CL
  - MIN-VCP