- 教材讨论
  - JH第4章第3节第1、2小节

问题1：greedy vs. local search

- greedy和local search有什么异同？

问题2：MIN-VCP

- 这个算法的基本思路是什么？
- 近似比是多少，为什么？
- 时间复杂度是多少？为什么？用什么数据结构来实现？

Algorithm 4.3.2.1. Input: A graph $G = (V, E)$.

Step 1: $C := \emptyset$ {during the computation $C \subseteq V$, and at the end C should contain a vertex cover};
$A := \emptyset$ {during the computation $A \subseteq E$ is a matching, and at the end A is a maximal matching};
$E' := E$ {during the computation $E' \subseteq E$, E' contains exactly the edges that are not covered by the actual C, and at the end $E' = \emptyset$}.

Step 2: **while** $E' \neq \emptyset$
do begin choose an arbitrary edge $\{u, v\}$ from E';
$C := C \cup \{u, v\}$;
$A := A \cup \{\{u, v\}\}$;
$E' := E' - \{$all edges incident to u or $v\}$
end

Output: C.

问题2： MIN-VCP (续)

- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为2的例子吗？

Algorithm 4.3.2.1. Input: A graph $G = (V, E)$.

Step 1: $C := \emptyset$ {during the computation $C \subseteq V$, and at the end C should contain a vertex cover};
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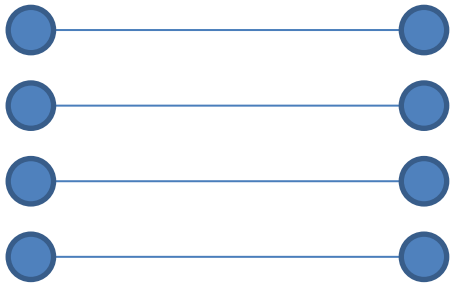
end

Output: C.

问题2： MIN-VCP (续)

- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为2的例子吗？



问题3：SCP

- 这个算法的基本思路是什么？
- 近似比是多少？
- 时间复杂度是多少？为什么？用什么数据结构来实现？

Algorithm 4.3.2.11.

Input: $(X, \mathcal{F})$, where X is a finite set, $\mathcal{F} \subseteq \mathcal{P}ot(X)$ such that $X = \bigcup_{Q \in \mathcal{F}} Q$.

Step 1: $C := \emptyset$ {during the computation $C \subseteq \mathcal{F}$ and at the end C is a set cover of $(X, \mathcal{F})$};

$U := X$ {during the computation $U \subseteq X$, $U = X - \bigcup_{Q \in C} Q$ for the actual C, and at the end $U = \emptyset$}.

Step 2: **while** $U \neq \emptyset$

do begin choose an $S \in \mathcal{F}$ such that $|S \cap U|$ is maximal;

$U := U - S$;

$C := C \cup \{S\}$

end

Output: C.

问题3： SCP (续)

- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为Ω(ln*n*)的例子吗？

Algorithm 4.3.2.11.

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Step 1: $C := \emptyset$ {during the computation $C \subseteq \mathcal{F}$ and at the end C is a set cover of $(X, \mathcal{F})$};

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do begin choose an $S \in \mathcal{F}$ such that $|S \cap U|$ is maximal;

$U := U - S$;

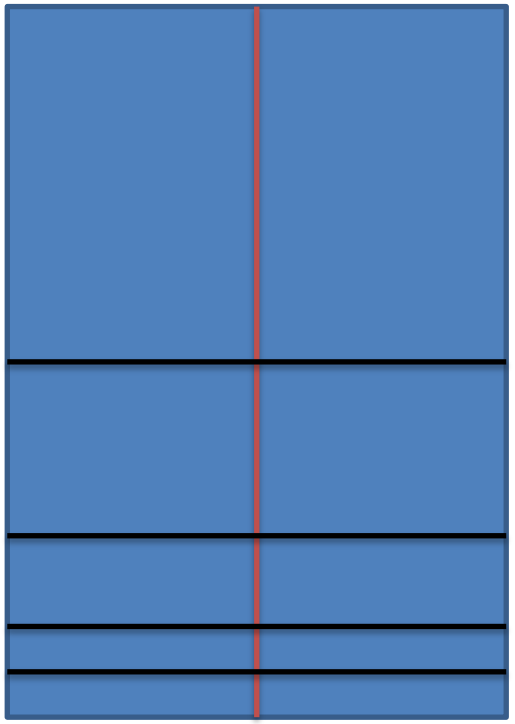
$C := C \cup \{S\}$

end

Output: C.

问题3： SCP (续)

- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为Ω(ln*n*)的例子吗？
  - $U = \{ (x,y) \mid 1 \le x,y \le n \}$
  - $S_1 = \{ (x,y) \mid x \le n/2 \}$
  - $S_2 = \{ (x,y) \mid x > n/2 \}$
  - $T_1 = \{ (x,y) \mid n/2 < y \le n \}$
  - $T_2 = \{ (x,y) \mid n/4 < y \le n/2 \}$
  - $T_3 = \{ (x,y) \mid n/8 < y \le n/4 \}$
  - ……



问题3：SCP (续)

- 和MIN-VCP相比，SCP到底难在什么地方？

问题4：MAX-CUT

- 这个算法的基本思路是什么？
- 近似比是多少，为什么？
- 时间复杂度是多少？为什么？用什么数据结构来实现？

Algorithm 4.3.3.1.

Input: A graph $G = (V, E)$.
Step 1: $S = \emptyset$
{the cut is considered to be $(S, V - S)$; in fact S can be chosen arbitrarily in this step};
Step 2: **while** there exists such a vertex $v \in V$ that the movement of v from one side of the cut $(S, V - S)$ to the other side of $(S, V - S)$ increases the cost of the cut.
do begin take a $u \in V$ whose movement from one side of $(S, V - S)$ to the other side of $(S, V - S)$ increases the cost of the cut, and move this u to the other side.
end
Output: $(S, V - S)$.

问题4： MAX-CUT (续)

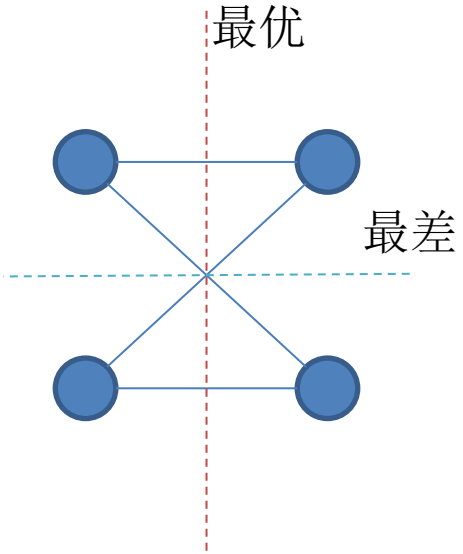
- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为2的例子吗？

Algorithm 4.3.3.1.

Input: A graph $G = (V, E)$.
Step 1: $S = \emptyset$
{the cut is considered to be $(S, V - S)$; in fact S can be chosen arbitrarily in this step};
Step 2: **while** there exists such a vertex $v \in V$ that the movement of v from one side of the cut $(S, V - S)$ to the other side of $(S, V - S)$ increases the cost of the cut.
do begin take a $u \in V$ whose movement from one side of $(S, V - S)$ to the other side of $(S, V - S)$ increases the cost of the cut, and move this u to the other side.
end
Output: $(S, V - S)$.

问题4： MAX-CUT (续)

- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为2的例子吗？



问题5：greedy和local search

- 任选1个问题，分别给出一种greedy算法和一种local search算法，并尽力给出近似比（或者给出一些坏例子）
  - longest simple path
  - MAX-SAT
  - MAX-CL