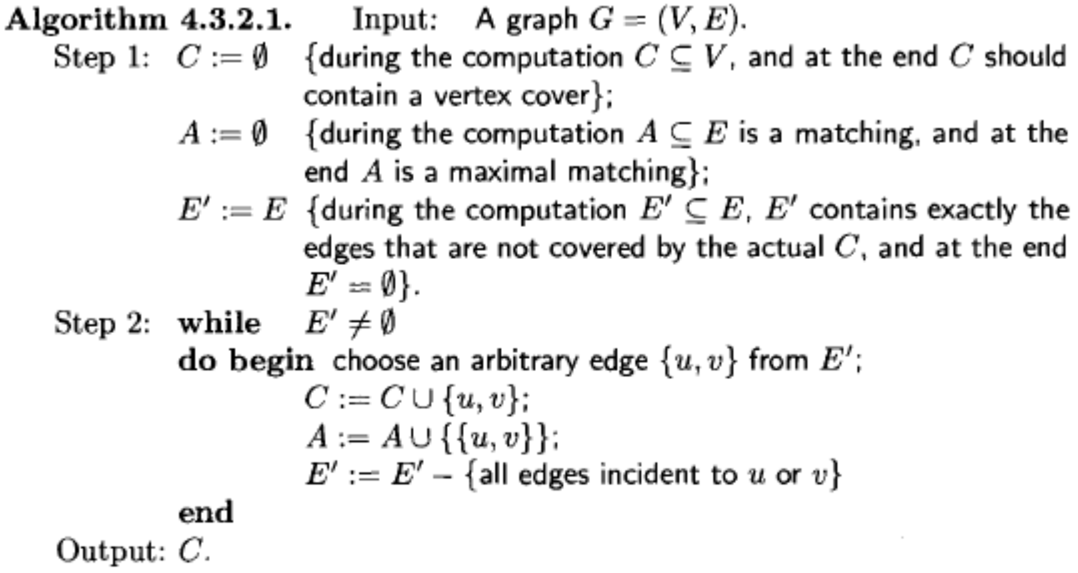
- 教材讨论
  - JH第4章第3节第1、2小节

问题1：greedy vs. local search

- greedy和local search有什么异同？

问题2： MIN-VCP

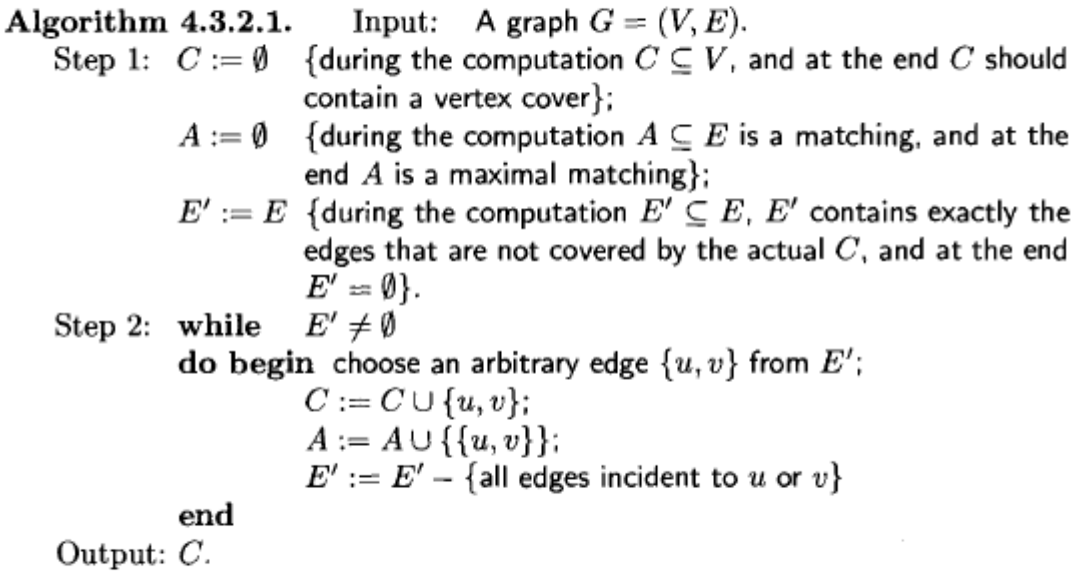
- 这个算法的基本思路是什么？
- 近似比是多少，为什么？
- 时间复杂度是多少？为什么？用什么数据结构来实现？



```
Algorithm 4.3.2.1.    Input:  A graph $G = (V, E)$.
    Step 1:  $C := \emptyset$   {during the computation $C \subseteq V$, and at the end $C$ should
                                 contain a vertex cover};
             $A := \emptyset$   {during the computation $A \subseteq E$ is a matching, and at the
                                 end $A$ is a maximal matching};
             $E' := E$          {during the computation $E' \subseteq E$, $E'$ contains exactly the
                                 edges that are not covered by the actual $C$, and at the end
                                 $E' = \emptyset$}.
    Step 2:  while              $E' \neq \emptyset$
             do begin  choose an arbitrary edge $\{u, v\}$ from $E'$;
                        $C := C \cup \{u, v\}$;
                        $A := A \cup \{\{u, v\}\}$;
                        $E' := E' - \{$all edges incident to $u$ or $v\}$
             end
    Output: $C$.
```

问题2： MIN-VCP (续)

- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为2的例子吗？



```
Algorithm 4.3.2.1.    Input:  A graph $G = (V, E)$.
    Step 1:  $C := \emptyset$   {during the computation $C \subseteq V$, and at the end $C$ should
                                 contain a vertex cover};
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             do begin  choose an arbitrary edge $\{u, v\}$ from $E'$;
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                       $A := A \cup \{\{u, v\}\}$;
                       $E' := E' - \{$all edges incident to $u$ or $v\}$
             end
    Output: $C$.
```
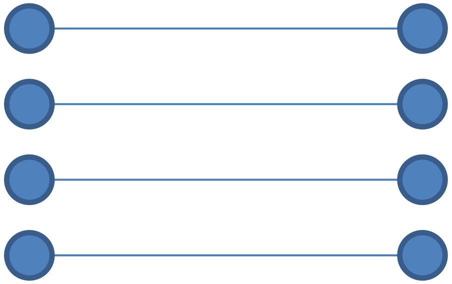
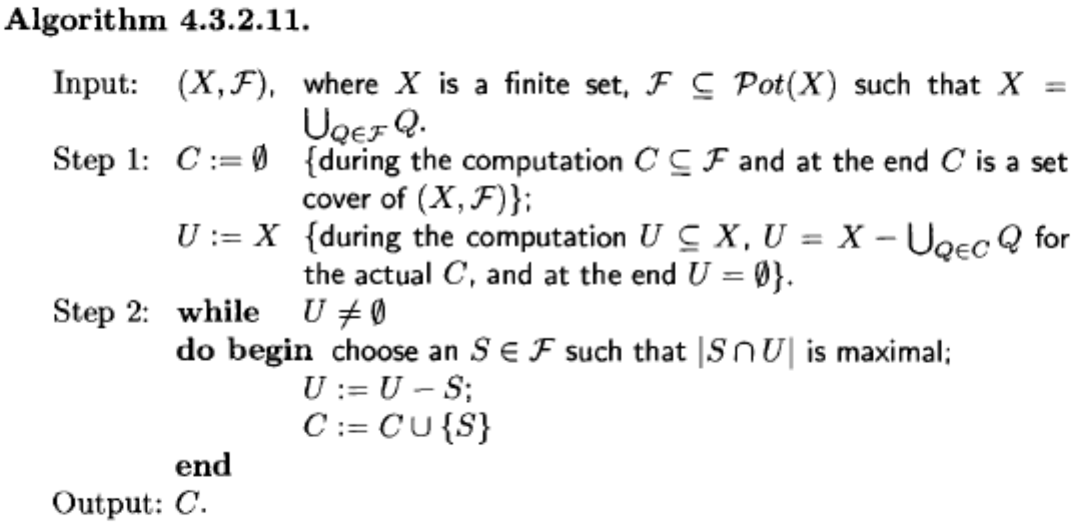

问题2： MIN-VCP (续)

- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为2的例子吗？



问题3：SCP

- 这个算法的基本思路是什么？
- 近似比是多少？
- 时间复杂度是多少？为什么？用什么数据结构来实现？

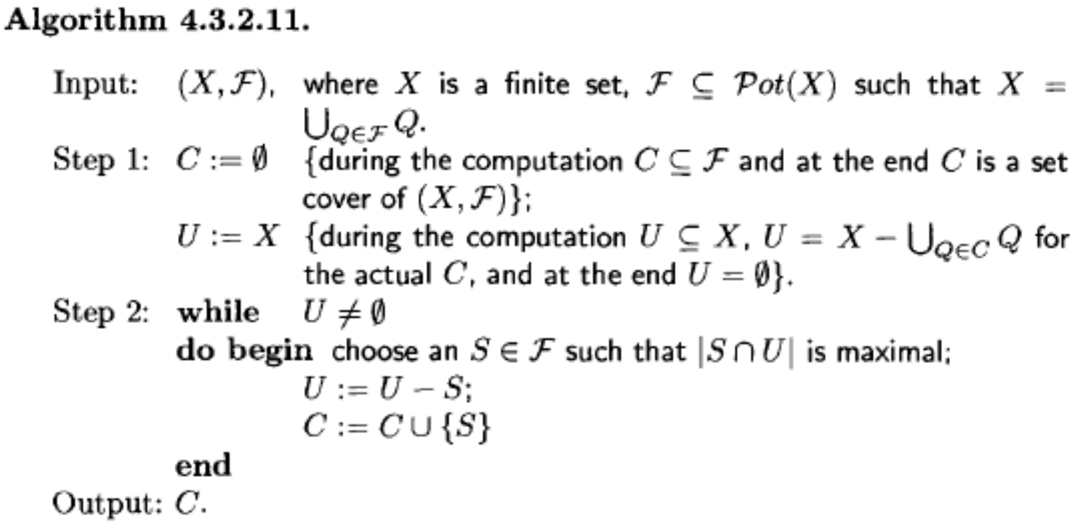


Algorithm 4.3.2.11.

```
Input:   $(X, \mathcal{F})$, where $X$ is a finite set, $\mathcal{F} \subseteq \mathcal{P}ot(X)$ such that $X = \bigcup_{Q \in \mathcal{F}} Q$.
Step 1:  $C := \emptyset$  {during the computation $C \subseteq \mathcal{F}$ and at the end $C$ is a set
                           cover of $(X, \mathcal{F})$};
         $U := X$  {during the computation $U \subseteq X$, $U = X - \bigcup_{Q \in C} Q$ for
                    the actual $C$, and at the end $U = \emptyset$}.
Step 2:  while  $U \neq \emptyset$
         do begin choose an $S \in \mathcal{F}$ such that $|S \cap U|$ is maximal;
                  $U := U - S$;
                  $C := C \cup \{S\}$
         end
Output: $C$.
```

问题3：SCP (续)

- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为Ω(ln*n*)的例子吗？

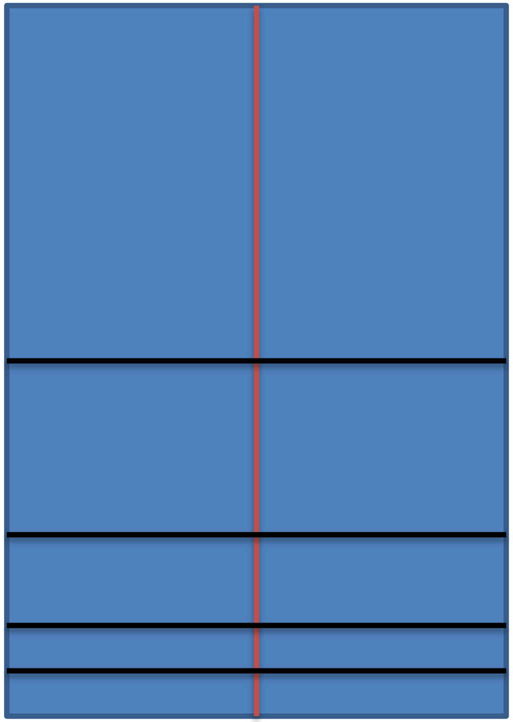


Algorithm 4.3.2.11.

```
Input:   $(X,\mathcal{F})$, where $X$ is a finite set, $\mathcal{F} \subseteq \mathcal{P}ot(X)$ such that $X = \bigcup_{Q\in\mathcal{F}} Q$.
Step 1:  $C := \emptyset$  {during the computation $C \subseteq \mathcal{F}$ and at the end $C$ is a set
                            cover of $(X,\mathcal{F})$};
         $U := X$  {during the computation $U \subseteq X$, $U = X - \bigcup_{Q\in C} Q$ for
                    the actual $C$, and at the end $U = \emptyset$}.
Step 2:  while $U \neq \emptyset$
         do begin choose an $S \in \mathcal{F}$ such that $|S \cap U|$ is maximal;
                  $U := U - S$;
                  $C := C \cup \{S\}$
         end
Output: $C$.
```

问题3：SCP (续)

- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为$\Omega(\ln n)$的例子吗？
  - $U = \{ (x,y) \mid 1 \le x,y \le n \}$
  - $S_1 = \{ (x,y) \mid x \le n/2 \}$
  - $S_2 = \{ (x,y) \mid x > n/2 \}$
  - $T_1 = \{ (x,y) \mid n/2 < y \le n \}$
  - $T_2 = \{ (x,y) \mid n/4 < y \le n/2 \}$
  - $T_3 = \{ (x,y) \mid n/8 < y \le n/4 \}$
  - ……

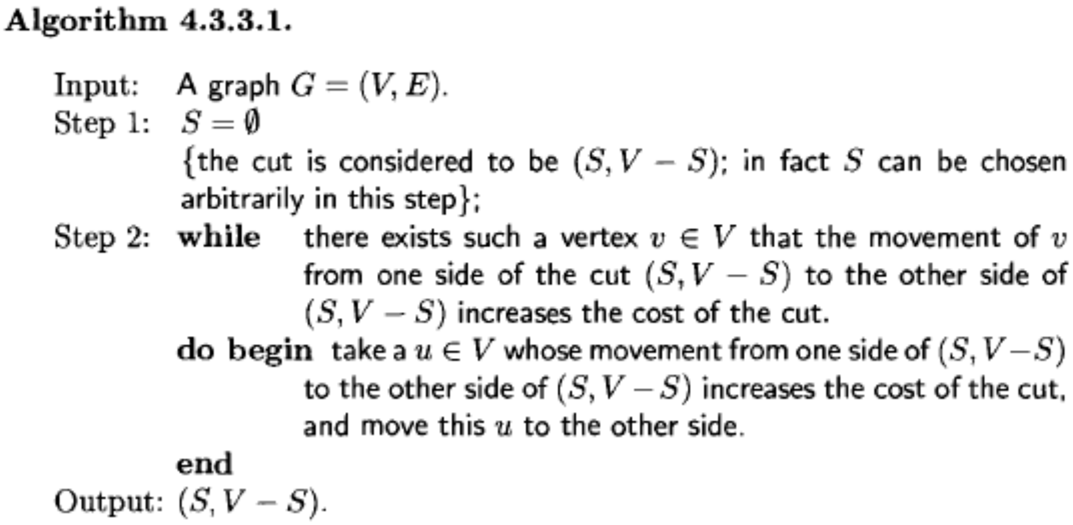


问题3：SCP (续)

- 和MIN-VCP相比，SCP到底难在什么地方？

问题4：MAX-CUT

- 这个算法的基本思路是什么？
- 近似比是多少，为什么？
- 时间复杂度是多少？为什么？用什么数据结构来实现？

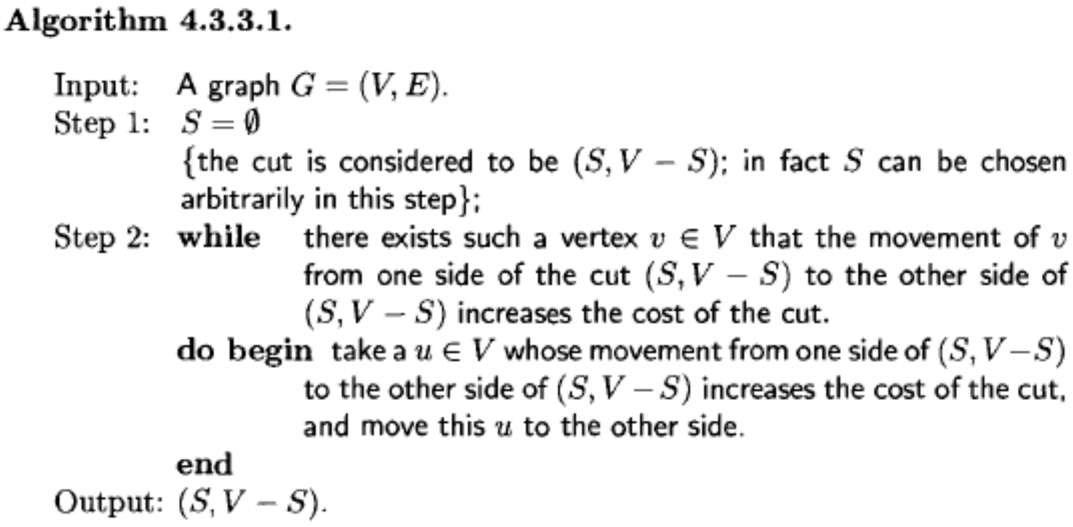


Algorithm 4.3.3.1.

```
Input:   A graph $G = (V, E)$.
Step 1:  $S = \emptyset$
         {the cut is considered to be $(S, V - S)$; in fact $S$ can be chosen
         arbitrarily in this step};
Step 2:  while    there exists such a vertex $v \in V$ that the movement of $v$
                  from one side of the cut $(S, V - S)$ to the other side of
                  $(S, V - S)$ increases the cost of the cut.
         do begin take a $u \in V$ whose movement from one side of $(S, V - S)$
                  to the other side of $(S, V - S)$ increases the cost of the cut,
                  and move this $u$ to the other side.
         end
Output:  $(S, V - S)$.
```

问题4： MAX-CUT (续)

- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为2的例子吗？

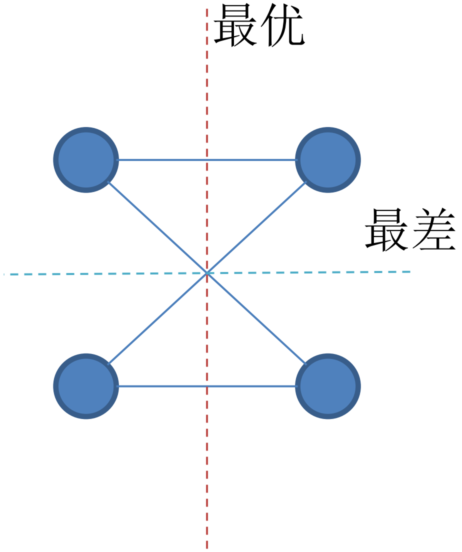


Algorithm 4.3.3.1.

```
Input:   A graph $G = (V, E)$.
Step 1:  $S = \emptyset$
         {the cut is considered to be $(S, V - S)$; in fact $S$ can be chosen
         arbitrarily in this step};
Step 2:  while    there exists such a vertex $v \in V$ that the movement of $v$
                  from one side of the cut $(S, V - S)$ to the other side of
                  $(S, V - S)$ increases the cost of the cut.
         do begin take a $u \in V$ whose movement from one side of $(S, V - S)$
                  to the other side of $(S, V - S)$ increases the cost of the cut,
                  and move this $u$ to the other side.
         end
Output:  $(S, V - S)$.
```

问题4： MAX-CUT (续)

- 你能构造出一些近似比为1的例子吗？
- 你能构造出一些近似比为2的例子吗？



问题5：greedy和local search

- 任选1个问题，分别给出一种greedy算法和一种local search算法，并尽力给出近似比（或者给出一些坏例子）
  - longest simple path
  - MAX-SAT
  - MAX-CL