Distances in Hamming space

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Metric space (M, d)

$d: M \times M \to \mathbb{R}$

1. d(x, y) = 0 iff x = y
2. d(x, y) = d(y, x)
3. d(x, z) <= d(x, y) + d(y, z)

Corollary: d(x, y) >= 0

Hamming space

$\{0, 1\}^N$

p-norm: $\|x\|_p = (|x_1|^p + |x_2|^p + \cdots + |x_n|^p)^{\frac{1}{p}}.$

- $p < 1$: Subadditivity does not hold
- $p = 1$: Manhattan distance -> Hamming distance
- $p = 2$: Euclidean distance
- $p \to +\infty$: Chebyshev distance

- (|x1| + |x2|^2 + |x3|^3 + |x4|^4 + … + |xn|^n)^k

Levenshtein/edit distance

- Insertion
- Deletion
- Substitution
- Less than or equal to hamming distance

Distances in undirected connected graph

- Complete graph: Discrete metric, d(x, y) = 1
- Hypercube: Hamming distance
- Ring: d(x, y) = min { x - y, y - x } (mod 2^N)

- Random maze

Thanks!