

计算机问题求解 — 论题1-11

- 算法方法

课程研讨

- DH第4章

问题1：搜索和遍历的应用

An arithmetic expression formed by non-negative integers and the standard unary operation “ $-$ ” and the binary operations “ $+$ ”, “ $-$ ”, “ \times ”, and “ $/$ ”, can be represented by a binary tree as follows:

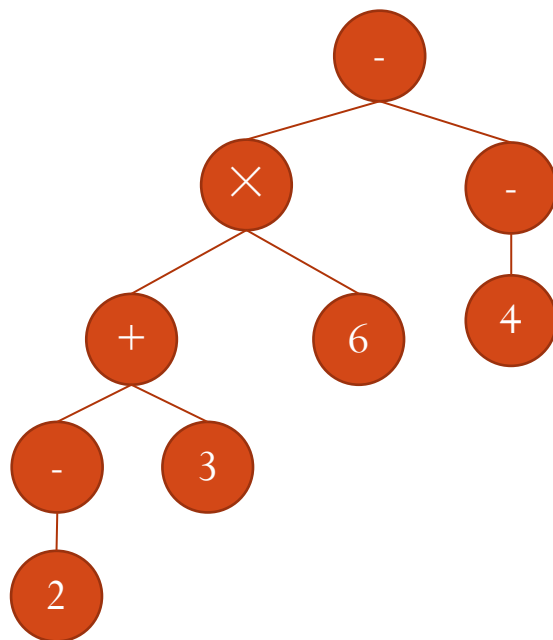
- An integer I is represented by a leaf containing I .
- The expression $-E$, where E is an expression, is represented by a tree whose root contains “ $-$ ” and its single offspring is the root of a subtree representing the expression E .
- The expression $E * F$, where E and F are expressions and “ $*$ ” is a binary operation, is represented by a tree whose root contains “ $*$ ”, its first offspring is the root of a subtree representing the expression E and its second offspring is the root of a subtree representing F .

Note that the symbol “ $-$ ” stands for both unary and binary operations, and the nodes of the tree containing this symbol may have outdegree either 1 or 2.

- 你能写出这个算式对应的树吗？
 $(-2+3) \times 6 - (-4)$

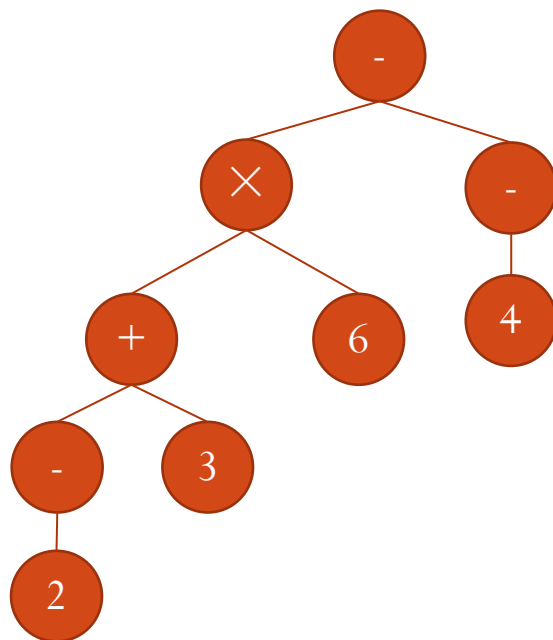
问题1：搜索和遍历的应用 (续)

- $(-2+3) \times 6 - (-4)$



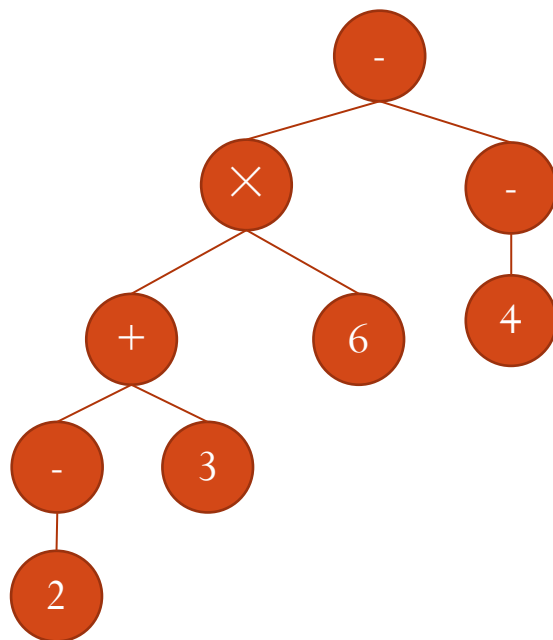
问题1：搜索和遍历的应用 (续)

Design an algorithm that checks whether a given tree represents an arithmetic expression.



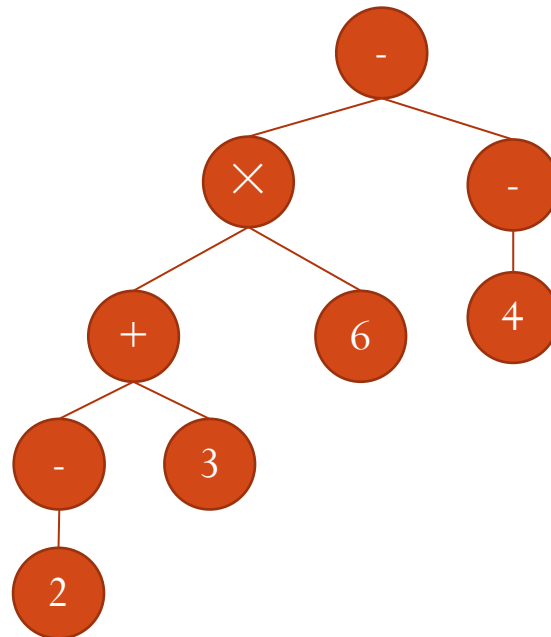
问题1：搜索和遍历的应用 (续)

Design an algorithm that calculates the value of an arithmetic expression, given its tree representation. Note that division by zero is undefined.



问题1：搜索和遍历的应用 (续)

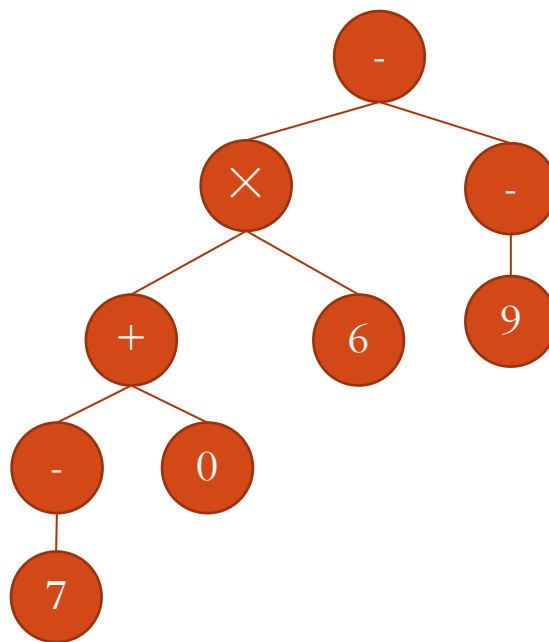
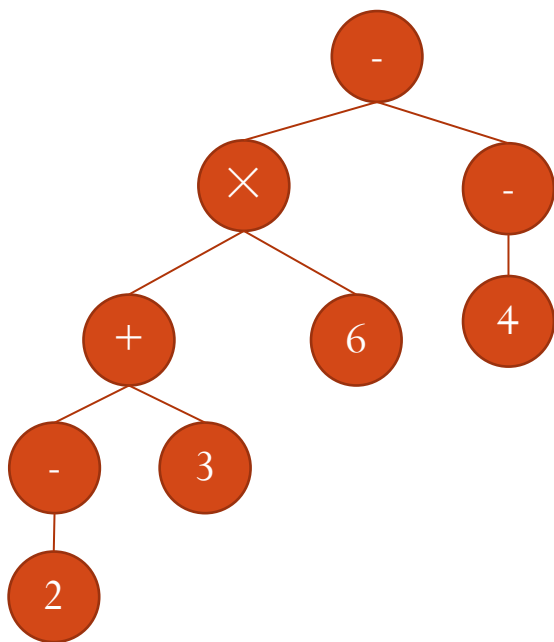
Extend your algorithm to first print the expression represented by the input tree, followed by the equality sign “=” and its evaluation. The printed expression should be fully parenthesized, i.e., a pair of matching parentheses should embrace every application of a binary operation.



问题1：搜索和遍历的应用 (续)

We say that two arithmetic expressions E and F are *isomorphic*, if E can be obtained from F by replacing some non-negative integers by others. For example, the expressions $(2 + 3) \times 6 - (-4)$ and $(7 + 0) \times 6 - (-9)$ are isomorphic, but none of them is isomorphic to any of $(-2 + 3) \times 6 - (-4)$ and $(7 + 0) + 6 - (-9)$.

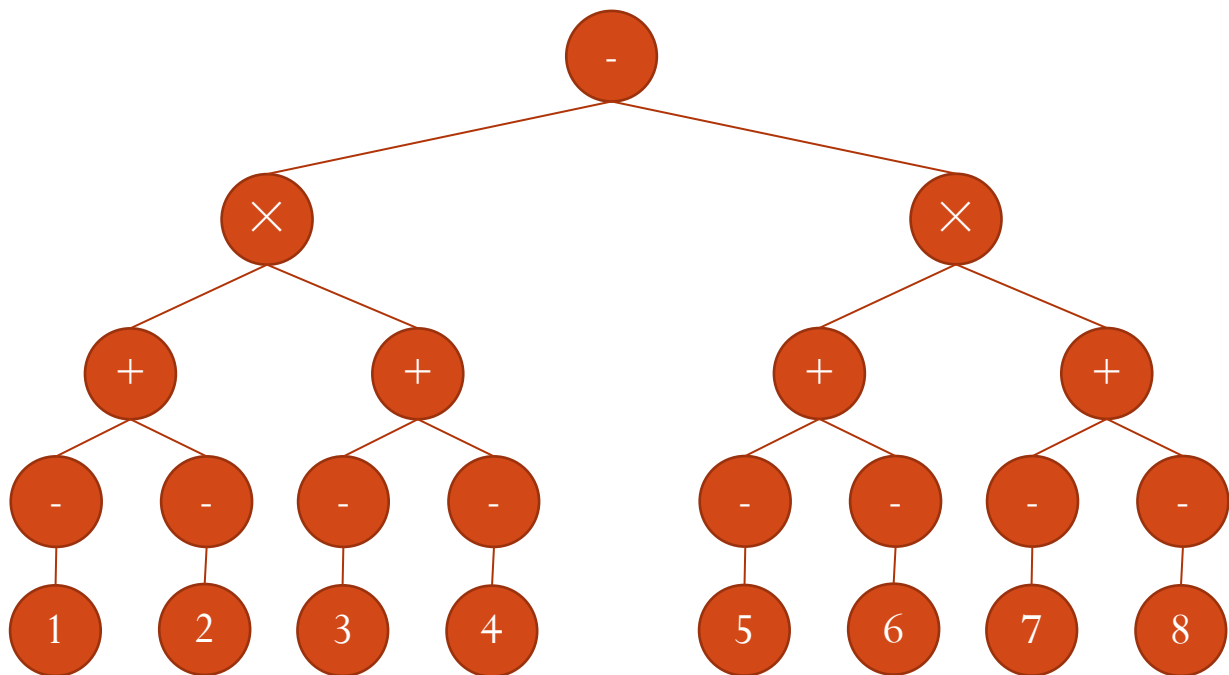
Design an algorithm that checks whether two expressions are isomorphic, given their tree representation.



问题1：搜索和遍历的应用 (续)

An expression E is said to be *balanced*, if every binary operation in it is applied to two isomorphic expressions. For example, the expressions -5 , $(1 + 2) * (3 + 5)$ and $((-3)/(-4))/((-1)/(-100))$ are balanced, while $12 + (3 + 2)$ and $3 * (-3)$ are not.

Design an algorithm that checks whether an expression is balanced, given its tree representation.

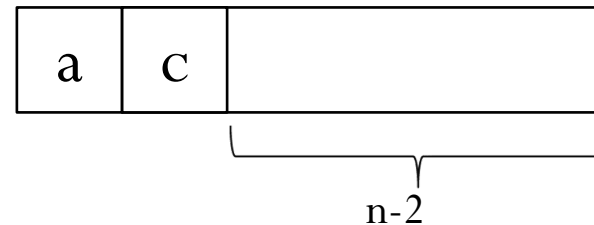
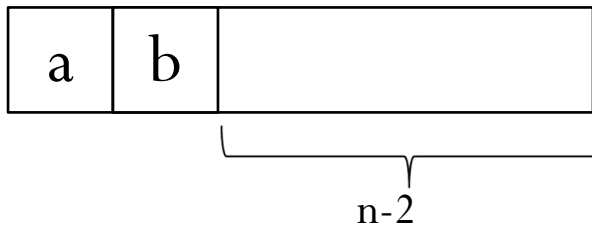
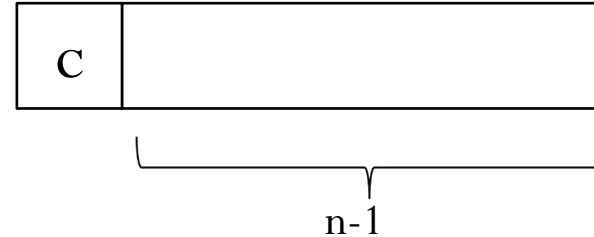
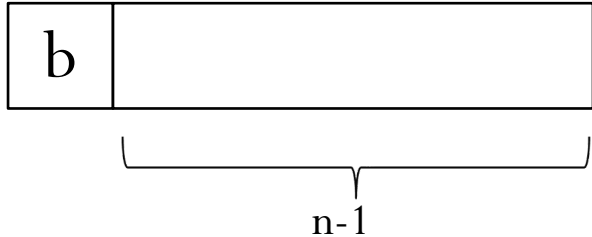


Number of Valid Strings

- String to be transmitted on the channel
 - Length n
 - Consisting of symbols 'a', 'b', 'c'
 - If "aa" exists, cannot be transmitted
 - E.g. strings of length 2: 'ab', 'ac', 'ba', 'bb', 'bc', 'ca', 'cc', 'cb'
- Number of valid strings?

Divide and Conquer

- $f(n) = 2f(n-1) + 2f(n-2), n > 2$
- $f(1) = 3, f(2) = 8$



问题2：算法方法的应用

- 组队拔河问题：学校组织拔河比赛，每班组一个队参加，人数不限，但要求总体重不超过300斤，如何组队最优？

	体重 (w)	力量 (P)
悟空	90	240
八戒	140	270
唐僧	100	210
白龙马	150	280
悟净	120	240

问题2： 算法方法的应用 (续)

- 你理解穷举搜索法了吗？
- 你能用穷举搜索法找到最优组队吗？
- 请写出伪代码

	体重 (w)	力量 (P)
悟空	90	240
八戒	140	270
唐僧	100	210
白龙马	150	280
悟净	120	240

问题2： 算法方法的应用 (续)

- 你能改进穷举搜索法，使其避免检查很多明显不可行的组队吗？
- 请写出改进后的伪代码

	体重 (w)	力量 (P)
悟空	90	240
八戒	140	270
唐僧	100	210
白龙马	150	280
悟净	120	240

问题2： 算法方法的应用 (续)

- 你理解贪心法了吗？
- 你能用贪心法找到最优/较优组队吗？
 - 你能想到几种贪心策略（从什么角度“贪”）？
- 请写出伪代码

	体重 (w)	力量 (P)
悟空	90	240
八戒	140	270
唐僧	100	210
白龙马	150	280
悟净	120	240

问题2： 算法方法的应用 (续)

- 你理解动态规划法了吗？
- 你能用动态规划法找到最优组队吗？
- 请写出伪代码

	体重 (w)	力量 (P)
悟空	90	240
八戒	140	270
唐僧	100	210
白龙马	150	280
悟净	120	240

问题3： Making Change

- Problem: with certain systems of coinage, how to pay a given amount using the smallest possible number of coins
- 贪心算法
- 动态规划

Subproblems

- Suppose the currency we are using has available coins of n different denominations, and a coin of denomination i has d_i units. The amount to be paid is N .
- One subproblem can be represented as $[i, j]$, for which the result is the minimum number of coins required to pay an amount of j units, using only coins of denominations 1 to i .
- The solution of the problem of making change is the result of subproblem $[n, N]$ (as $c[n, N]$)

Dependency of Subproblems

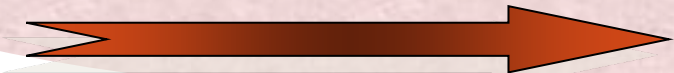
- $c[i,0]$ is 0 for all i
- When we are to pay an amount j using coins of denominations 1 to i , we have two choices:
 - No coins of denomination i is used: $c[i-1, j]$
 - One coins of denomination i is used: $1+c[i, j-d_i]$
- So, $c[i, j] = \min (c[i-1, j], 1+c[i, j-d_i])$

Data Structure

Define a array $\text{coin}[1..n, 0..N]$ for all $c[i, j]$

an example

	0	1	2	3	4	5	6	7	8
$d_1=1$	0	1	2	3	4	5	6	7	8
$d_2=4$	0	1	2	3	1	2	3	4	2
$d_3=6$	0	1	2	3	1	2	1	2	2



direction of computation