计算机问题求解 – 论题2-2

- 组合与计数

  课程研讨

  - CS第1章

问题1：加法和乘法

- 你是如何理解加法原理和乘法原理的？
  在算法分析中，如何利用这两个原理来计数？

Principle 1.2 (Sum Principle) *If a finite set S has been partitioned into blocks, then the size of S is the sum of the sizes of the blocks.*

Principle 1.3 (Product Principle) *The size of a union of m disjoint sets, each of size n, is mn.*

问题1：加法和乘法 (续)

```
for i = 2 to n
    j=i
    while j ≥ 2 and A[j] < A[j − 1]
        exchange A[j] and A[j − 1]
        j − −
```

- 这个算法的关键操作是什么？
- 最坏情况下，这个关键操作需要运行多少次？
- 你用了哪条原理？这里的集合是什么？

问题1：加法和乘法 (续)

```
for i = 1 to n − 1
        minval = A[i]
        minindex = i
        for j = i to n
            if (A[j] < minval)
                minval = A[j]
                minindex = j
        exchange A[i] and A[minindex]
```

- 这个算法的关键操作是什么？
- 最坏情况下，这个关键操作需要运行多少次？
- 你用了哪条原理？这里的集合是什么？

问题1：加法和乘法 (续)

In how many ways may a ten person club select a president and a secretary-treasurer from among its members?

- 你用了哪条原理？这里的集合是什么？

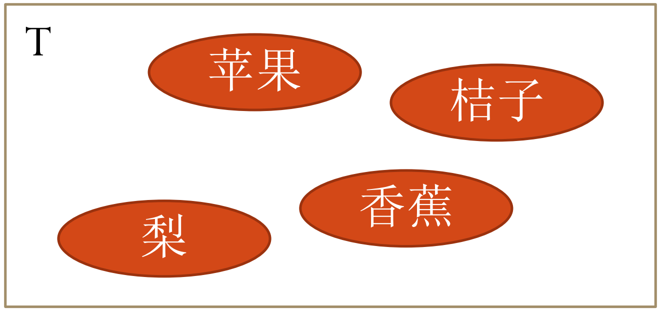
问题1：加法和乘法 (续)

The "Pile High Deli" offers a "simple sandwich" consisting of your choice of one of five different kinds of bread with your choice of butter or mayonnaise or no spread, one of three different kinds of meat, and one of three different kinds of cheese, with the meat and cheese "piled high" on the bread. In how many ways may you choose a simple sandwich?

- 你用了哪条原理？这里的集合是什么？

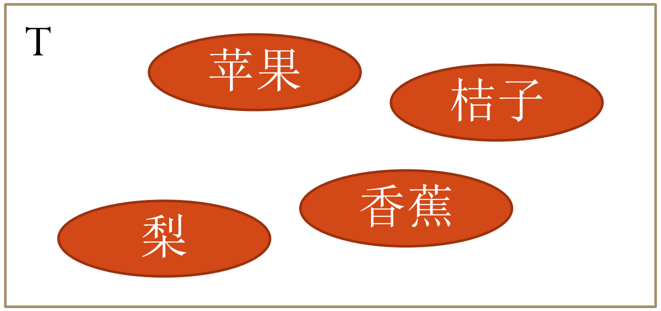
问题2：列表、置换和子集

- 列表和集合有什么区别？
- 列表的数学本质是什么？与集合有什么联系？
  给定一个集合，你能构造多少种列表？
- 置换的数学本质是什么？与集合、列表各有什么联系？
  给定一个集合，你能构造多少种置换？



问题2：列表、置换和子集 (续)

- k-元素置换的数学本质是什么？
  给定一个集合，你能构造多少种k-元素置换？
- k-元素子集的数学本质是什么？
  给定一个集合，你能构造多少种k-元素子集？



问题2：列表、置换和子集 (续)

We are making a list of participants in a panel discussion on allowing alcohol on campus. They will be sitting behind a table in the order in which we list them. There will be four administrators and four students. In how many ways may we list them if the administrators must sit together in a group and the students must sit together in a group? In how many ways may we list them if we must alternate students and administrators?

- 思考：你用到了k-元素置换还是k-元素子集？

问题2：列表、置换和子集 (续)

We are choosing participants for a panel discussion allowing on allowing alcohol on campus. We have to choose four administrators from a group of ten administrators and four students from a group of twenty students. In how many ways may we do this?

- 思考：你用到了k-元素置换还是k-元素子集？

问题2：列表、置换和子集 (续)

We are making a list of participants in a panel discussion on allowing alcohol on campus. They will be sitting behind a table in the order in which we list them. There will be four administrators chosen from a group of ten administrators and four students chosen from a group of twenty students. In how many ways may we choose and list them if the administrators must sit together in a group and the students must sit together in a group? In how many ways may we choose and list them if we must alternate students and administrators?

- 思考：你用到了k-元素置换还是k-元素子集？

问题2：列表、置换和子集 (续)

A basketball team has 12 players. However, only five players play at any given time during a game. In how may ways may the coach choose the five players? To be more realistic, the five players playing a game normally consist of two guards, two forwards, and one center. If there are five guards, four forwards, and three centers on the team, in how many ways can the coach choose two guards, two forwards, and one center? What if one of the centers is equally skilled at playing forward?

- 思考：你用到了k-元素置换还是k-元素子集？

问题3：双射

- 双射在counting中有什么作用?

- Determine the number of triangles

```
(1)  trianglecount = 0
(2)  for i = 1 to n
(3)      for j = i + 1 to n
(4)          for k = j + 1 to n
(5)              if points i, j, and k are not collinear
(6)                  trianglecount = trianglecount +1
```

- 这个算法的关键操作是什么?
- 最坏情况下，这个关键操作需要运行多少次?
- 这里的双射是什么?

问题3：双射 (续)

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{k}\binom{n-k}{j} = \binom{n}{j}\binom{n-j}{k}$$

- 你能利用双射证明这些等式吗？
（在证明中，你用到的双射是什么？）

问题4：等价关系与除法

- 等价关系在counting中有什么作用？

问题4：等价关系与除法

- 等价关系在counting中有什么作用？

Principle 1.7 (Quotient Principle.) *If we can partition a set of size p into q blocks of size r, then $q = p/r$.*

问题4：等价关系与除法 (续)

Exercise 1.4-2 When four people sit down at a round table to play cards, two lists of their four names are equivalent as seating charts if each person has the same person to the right in both lists[9]. (The person to the right of the person in position 4 of the list is the person in position 1). We will use Theorem 1.5 to count the number of possible ways to seat the players. We will take our set S to be the set of all 4-element permutations of the four people, i.e., the set of all lists of the four people.

- 你能利用等价关系求解吗?
- 4!/4

问题4：等价关系与除法 (续)

In how many ways may n men and n women be seated around a table alternating gender?

- 你能利用等价关系求解吗？

问题4：等价关系与除法 (续)

In how many ways may n men and n women be seated around a table alternating gender?

- 你能利用等价关系求解吗？
  $2n!n!/2n=n!(n-1)!$

问题4：等价关系与除法 (续)

Exercise 1.4-3 We wish to count the number of ways to attach n distinct beads to the corners of a regular n-gon (or string them on a necklace). We say that two lists of the n beads are equivalent if each bead is adjacent to exactly the same beads in both lists. (The first bead in the list is considered to be adjacent to the last.)

- 你能利用两种不同的等价关系求解吗？

问题4：等价关系与除法 (续)

Exercise 1.4-3 We wish to count the number of ways to attach n distinct beads to the corners of a regular n-gon (or string them on a necklace). We say that two lists of the n beads are equivalent if each bead is adjacent to exactly the same beads in both lists. (The first bead in the list is considered to be adjacent to the last.)

- 你能利用两种不同的等价关系求解吗？
  $n!/2n$、$(n-1)!/2$

问题4：等价关系与除法 (续)

In how many ways may n red checkers and $n+1$ black checkers be arranged in a circle?

- 你能利用等价关系求解吗？

问题4：等价关系与除法 (续)

In how many ways may n red checkers and $n+1$ black checkers be arranged in a circle?

- 你能利用等价关系求解吗？
  $(2n+1)!/(2n+1)(n+1)!n!$

问题4：等价关系与除法 (续)

In how many ways three identical red apples and two identical golden apples may be lined up in a line?

- 你能利用等价关系求解吗？

问题4：等价关系与除法 (续)

In how many ways three identical red apples and two identical golden apples may be lined up in a line?

- 你能利用等价关系求解吗？
  5!/(2!3!)

问题4：等价关系与除法 (续)

Exercise 1.4-5 We have k books to arrange on the n shelves of a bookcase. The order in which the books appear on a shelf matters, and each shelf can hold all the books. We will assume that as the books are placed on the shelves they are moved as far to the left as they will go so that all that matters is the order in which the books appear and not the actual places where the books sit. When book i is placed on a shelf, it can go between two books already there or to the left or right of all the books on that shelf.

- 你能利用等价关系求解吗？

问题4：等价关系与除法 (续)

Exercise 1.4-5 We have k books to arrange on the n shelves of a bookcase. The order in which the books appear on a shelf matters, and each shelf can hold all the books. We will assume that as the books are placed on the shelves they are moved as far to the left as they will go so that all that matters is the order in which the books appear and not the actual places where the books sit. When book i is placed on a shelf, it can go between two books already there or to the left or right of all the books on that shelf.

- 你能利用等价关系求解吗？

$$\frac{(n+k-1)!}{(n-1)!}$$

问题4：等价关系与除法 (续)

Exercise 1.4-6 How many k-element multisets can we choose from an n-element set?

- 你能利用等价关系求解吗？

问题4：等价关系与除法 (续)

Exercise 1.4-6 How many k-element multisets can we choose from an n-element set?

- 你能利用等价关系求解吗？

$$\frac{n^{\overline{k}}}{k!}$$