反馈与讨论

2014-4-3

15. A tennis club has $2n$ members. We want to pair up the members by twos for singles matches. In how many ways can we pair up all the members of the club? Suppose that in addition to specifying who plays whom, we also determine who serves first for each pairing. Now in how many ways can we specify our pairs?

1. $\binom{2n}{2} * \binom{2n-2}{2} * \binom{2n-4}{2} * \ldots * \binom{4}{2} * \binom{2}{2} / n! = \frac{(2n)!}{2^n n!}$.

2. $\frac{(2n)!}{2^n n!} * 2^n = \frac{(2n)!}{n!}$.

14. Give at least two proofs that

$$\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}.$$

(1) By def. of n choose k.
(2) By explanation: we choose j # of item for purpose 1 and k-j # of item for purpose 2 from total n items.

8. The formula for the number of multisets is $(n + k - 1)!$ divided by a product of two other factorials. We want to use the quotient principle to explain why this formula counts multisets. The formula for the number of multisets is also a binomial coefficient, so it should have an interpretation that involves choosing k items from $n + k - 1$ items. The parts of the problem that follow lead us to these explanations.

   a. In how many ways can you place k red checkers and $n - 1$ black checkers in a row?

   b. How can you relate the number of ways of placing k red checkers and $n - 1$ black checkers in a row to the number of k-element multisets of an n-element set (the set $\{1, 2, \ldots, n\}$ to be specific)?

   c. How can you relate the choice of k items out of $n + k - 1$ items to the placement of red and black checkers, as in parts a and b? Think about how this relates to placing k identical books and $n - k$ identical blocks of wood in a row.

4. *Multiset.* A multiset is similar to a set, except each item can appear multiple times. We can specify a *multiset* chosen from a set S by saying how many times each of its elements occurs.

5. *Choosing* k-*element multisets.* The number of k-element multisets that can be chosen from an n-element set is

$$\frac{(n+k-1)!}{k!\,(n-1)!} = \binom{n+k-1}{k}.$$

a. For (k+n-1) different items, there are (k+n-1)! ways. But for k red and n-1 black ones k!(n-1)! ways are equivalent, the answer if (k+n-1)!/(k!(n-1)!).
b. n-1 black checkers make n spaces for the k red checkers.
c. Choose k places for the red checkers from (k+n-1) places

9. How many solutions to the equation $x_1 + x_2 + \cdots + x_n = k$ are there with each x_i a nonnegative integer?

10. How many solutions to the equation $x_1 + x_2 + \cdots + x_n = k$ are there with each x_i a positive integer?

9. The answer is the # of k-element multisets from n-element set.

10. k items are to be partitioned into n groups. Each group cannot be empty. There are (k-1) spaces, choose (n-1) spaces from them.

PS. We can first let $x_i=1$; and then check the number of solutions for $x_1+x_2+\ldots+x_n = k-n$; now x_i is nonnegative.

15. Answer the following questions with either n^k, $n^{\underline{k}}$, $\binom{n}{k}$, or $\binom{n+k-1}{k}$.

a. In how many ways can k different candy bars be distributed to n people (with any person allowed to receive more than one bar)?

b. In how many ways can k different candy bars be distributed to n people (with nobody receiving more than one bar)?

c. In how many ways can k identical candy bars be distributed to n people (with any person allowed to receive more than one bar)?

d. In how many ways can k identical candy bars be distributed to n people (with nobody receiving more than one bar)?

e. How many one-to-one functions f are there from $\{1, 2, \ldots, k\}$ to $\{1, 2, \ldots, n\}$?

f. How many functions f are there from $\{1, 2, \ldots, k\}$ to $\{1, 2, \ldots, n\}$?

g. In how many ways can you choose a k-element subset from an n-element set?

h. How many k-element multisets can be formed from an n-element set?

i. In how many ways can the top k-ranking officials in the U.S. government be chosen from a group of n people? (We want an ordered list of the people, not a set.)

j. In how many ways can k pieces of candy (not necessarily of different types) be chosen from among n different types?

k. In how many ways can k children each choose one piece of candy (all of different types) from among n different types of candy?