

- 作业讲解
 - JH第3章练习3.4.2.1、3.4.2.2、3.5.2.3
 - 采用branch-and-bound方法解背包问题，并分别给出对你的方法有利与不利的输入

JH第3章练习3.5.2.3

- 证明参见
 - 《Solving satisfiability in less than 2^n steps》
 - <http://www.sciencedirect.com/science/article/pii/0166218X85900502>

采用branch-and-bound方法解背包问题

- 物品按性价比排序，逐个尝试（DFS）。
- 根据当前物品的性价比（是后续物品性价比上界）计算总额的上界，判断可否剪枝。

- 教材讨论
 - JH第3章第7节

问题1：用0-1规划建模

- 0-1 KP

$$\text{Maximize } \sum_{i=1}^n v_i x_i \text{ subject to } \sum_{i=1}^n w_i x_i \leq W, \quad x_i \in \{0, 1\}$$

问题1：用0-1规划建模

- multiple KP
 - n items and m knapsacks with capacities W_i

$$\text{maximize } \sum_{i=1}^m \sum_{j=1}^n p_j x_{ij}$$

$$\text{subject to } \sum_{j=1}^n w_j x_{ij} \leq W_i, \text{ for all } 1 \leq i \leq m$$

$$\sum_{i=1}^m x_{ij} \leq 1, \quad \text{for all } 1 \leq j \leq n$$

$$x_{ij} \in \{0, 1\} \quad \text{for all } 1 \leq j \leq n \text{ and all } 1 \leq i \leq m$$

问题1：用0-1规划建模 (续)

- 0-1 multidimensional KP
 - e.g., with both a volume limit and a weight limit

$$\begin{aligned} & \text{maximize } \sum_{j=1}^n p_j x_j \\ & \text{subject to } \sum_{j=1}^n w_{ij} x_j \leq W_i, \text{ for all } 1 \leq i \leq m \\ & \quad x_j \in \{0, 1\} \end{aligned}$$

问题1：用0-1规划建模 (续)

- SCP

$$\text{minimize } \sum_{i=1}^m x_i$$

under the following n linear constraints

$$\sum_{j \in \text{Index}(k)} x_j \geq 1 \text{ for } k = 1, \dots, n.$$

问题1: 用0-1规划建模 (续)

- MS

$$\begin{aligned} & \text{minimize} && t \\ & \text{subject to} && \sum_{i \in M} x_{ij} = 1, \quad j \in J \\ & && \sum_{j \in J} x_{ij} p_{ij} \leq t, \quad i \in M \\ & && x_{ij} \in \{0, 1\}, \quad i \in M, j \in J \end{aligned}$$

问题1: 用0-1规划建模 (续)

- MAX-SAT

$$\text{maximize } \sum_{c \in \mathcal{C}} z_c$$

$$\text{subject to } \forall c \in \mathcal{C} : \sum_{i \in S_c^+} y_i + \sum_{i \in S_c^-} (1 - y_i) \geq z_c$$
$$\forall c \in \mathcal{C} : z_c \in \{0, 1\}$$
$$\forall i : y_i \in \{0, 1\}$$

问题1: 用0-1规划建模 (续)

- The **facility location problem** consists of a set of potential facility F that can be opened, and a set of cities C that must be serviced. The goal is to pick a subset of facilities to open, to minimize the sum of distances from each city to its nearest facility, plus the sum of opening costs of the facilities.

$$\begin{aligned} &\text{minimize} && \sum_{i \in F, j \in C} c_{ij} x_{ij} + \sum_{i \in F} f_i y_i \\ &\text{subject to} && \sum_{i \in F} x_{ij} = 1, && j \in C \\ &&& y_i - x_{ij} \geq 0, && i \in F, j \in C \\ &&& x_{ij} \in \{0, 1\}, && i \in F, j \in C \\ &&& y_i \in \{0, 1\}, && i \in F \end{aligned}$$

问题1: 用0-1规划建模 (续)

- WEIGHT-VCP

minimize

$$\sum_{i=1}^n c(v_i) \cdot x_i.$$

$$x_i \in \{0, 1\}$$

$$x_i + x_j \geq 1 \text{ for every } \{v_i, v_j\} \in E$$

问题1: 用0-1规划建模 (续)

- maximum matching

maximize

$$\sum_{e \in E} x_e$$

under the $|V|$ constraints

$$\sum_{e \in E(v)} x_e \leq 1 \text{ for every } v \in V,$$

and the following $|E|$ constraints

$$x_e \in \{0, 1\} \text{ for every } e \in E.$$

问题1: 用0-1规划建模 (续)

- WEIGHT-CL

$$\max \sum_{i=1}^n w_i x_i,$$

$$\text{s.t. } x_i + x_j \leq 1, \forall (i, j) \in \overline{E},$$

$$x_i \in \{0, 1\}, i = 1, \dots, n.$$

问题1: 用0-1规划建模 (续)

- TSP

$$\min z = \sum_{j=2}^{j=n} \sum_{i=1}^{j-1} c_{ij} x_{ij},$$

subject to: $x_{ij} = 0, 1, \quad (i = 1, \dots, j-1; j = 2, \dots, n)$
and the loop constraints

$$\sum_{i \in S} \sum_{j \in \bar{S}} x_{ij} \geq 2,$$

for all nonempty partitions (S, \bar{S}) such that if (S, \bar{S}) is considered (\bar{S}, S) is not.

问题2: rounding

- 什么是一个好的rounding?
 - The obtained rounded integral solution is a feasible solution.
 - The cost has not been changed too much.

问题2: rounding (续)

- WEIGHT-VCP可以怎样rounding? 得到的结果有多好?

minimize

$$\sum_{i=1}^n c(v_i) \cdot x_i.$$

$$x_i \in \{0, 1\}$$

$$x_i + x_j \geq 1 \text{ for every } \{v_i, v_j\} \in E$$

问题2: rounding (续)

- SCP(k)可以怎样rounding? 得到的结果有多好?

$$\text{minimize } \sum_{i=1}^m x_i$$

under the constraints

$$\sum_{h \in \text{Index}(a_j)} x_h \geq 1 \text{ for } j = 1, \dots, n,$$

$$x_i \in \{0, 1\} \text{ for } i = 1, \dots, m,$$

问题2: rounding (续)

- MAX-SAT可以怎样rounding? 你会分析结果的好坏吗?

$$\begin{aligned} &\text{maximize} && \sum_{c \in \mathcal{C}} z_c \\ &\text{subject to} && \forall c \in \mathcal{C} : \sum_{i \in S_c^+} y_i + \sum_{i \in S_c^-} (1 - y_i) \geq z_c \\ &&& \forall c \in \mathcal{C} : z_c \in \{0, 1\} \\ &&& \forall i : y_i \in \{0, 1\} \end{aligned}$$

- randomized rounding

问题2: rounding (续)

- 我们能用类似的方法处理WEIGHT-VCP吗?
 - 如果能: 请给出具体的做法
 - 如果不能: 请说明原因

minimize

$$\sum_{i=1}^n c(v_i) \cdot x_i.$$

$$x_i \in \{0, 1\}$$

$$x_i + x_j \geq 1 \text{ for every } \{v_i, v_j\} \in E$$

问题3：广义的relaxation

- relaxation并不限于LP，它是一种思想
 - 在更大范围内求解更容易的一个问题，再修正到原始范围
- 你能利用这个思想给出一个求最长哈密尔顿圈的近似算法吗？
 - 提示：哈密尔顿圈和匹配之间有什么关系？
- 偶数个顶点：哈密尔顿圈=2个匹配
 - 将找最长哈密尔顿圈松弛为找一个最大匹配（再补齐为哈密尔顿圈）
 - 得到的结果有多好？
- 奇数个顶点：哈密尔顿圈=3个匹配
 - 同理