

- 书面作业讲解
  - TC第15.1节练习1、3
  - TC第15.2节练习2、4
  - TC第15.3节练习3、5、6
  - TC第15.4节练习3、5
  - TC第15.5节练习1
  - TC第15章问题4

# TC第15.1节练习1

- 要重视数学归纳法的结构：
  - $n=0$ 时.....
  - 假设 $n \leq k$ 时成立，则 $n=k+1$ 时.....

# TC第15.2节练习2

- Give a recursive algorithm that actually performs the optimal matrix-chain multiplication.

```
MATRIX-CHAIN-MULTIPLY (A, s, i, j) {  
    if (i==j)  
        return Ai;  
    else if ((i+1)==j)  
        return Ai × Aj; //这步需要吗?  
    else  
        return MATRIX-CHAIN-MULTIPLY(A, s, i, s[i, j]) × MATRIX-CHAIN-MULTIPLY(A, s, s[i, j]+1, j);  
}
```

# TC第15.2节练习4

- Subproblem graph

- Vertex: Subproblem.

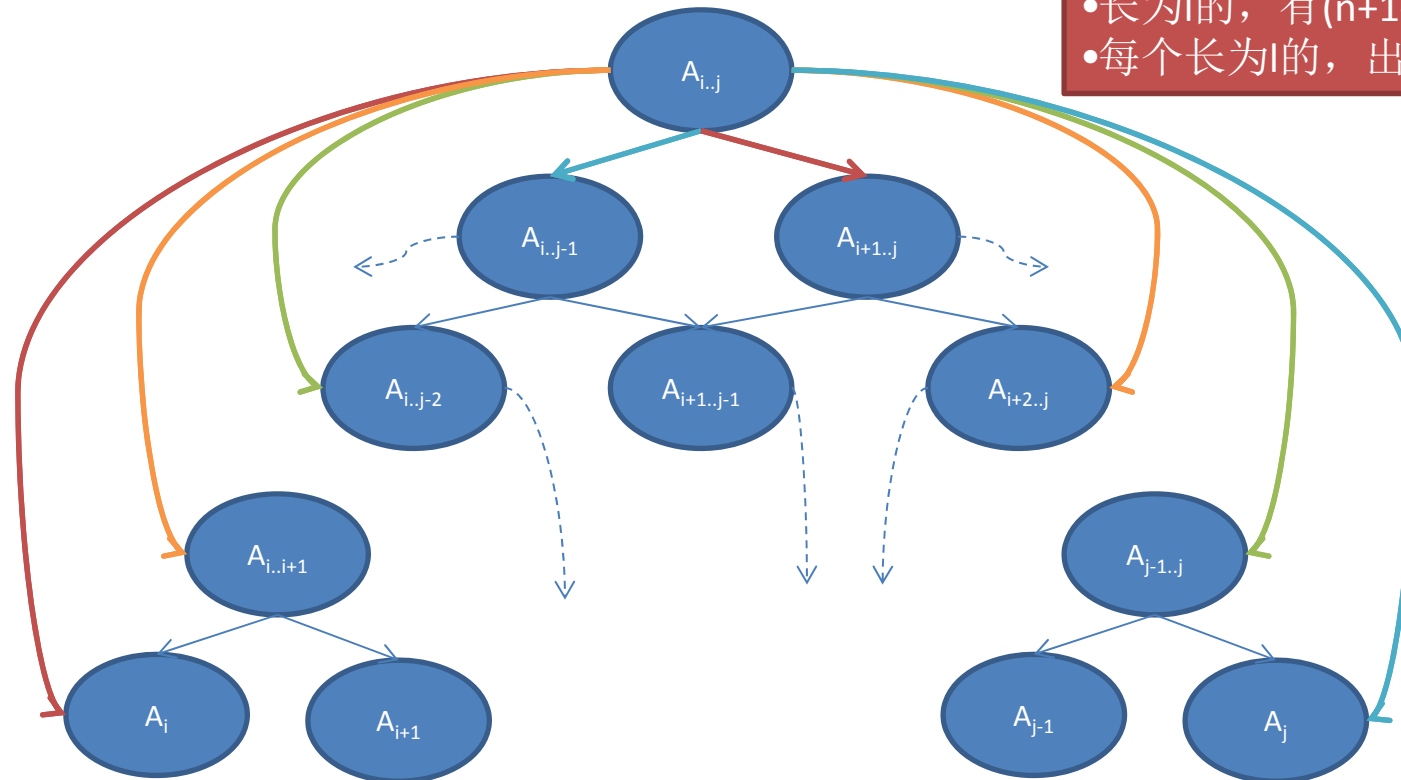
$$= n + \dots + 1 = n(n+1)/2$$

- Edge (x, y): Need a solution to y when solving x.

$$= \sum_l (n+1-l)2(l-1)$$

- 长为 $l$ 的, 有 $(n+1-l)$ 个

- 每个长为 $l$ 的, 出度为 $2(l-1)$



# TC第15.3节练习6

- A sequency of trades may entail a commission, which depends on the **number of trades** you make.
  - $c_k$ : 总共兑换k次的费用

# TC第15.4节练习5

- 方法1
  - $m(i) = \max_{1 \leq j < i} \{m[j]\} + 1, \text{ s.t. } x[j] \leq x[i]$
- 方法2
  - $Y \leftarrow X;$
  - $\text{SORT}(Y);$
  - $\text{LCS-LENGTH}(X, Y);$

1	5	2	4	9	7	8	5
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# TC第15章问题4

- Printing neatly
  - 子问题和最优子结构是什么？

Aaa bbb aaa bbb Aaa bbb aaa bbb Aaa bbb aaa bbb a  
bbb aaa bbb bbb aaa bbb Aaa bbb aaa bbb Aaa bb aaa  
bbb Aaa bbb aaa bbb Aaa bbb aaa bbb Aaa bb aaa bb  
Aaa bbb aaa bbb Aaa bbb aaa bbb Aaa bbb aaa bbb Aaa  
bbb aaa bbb Aaa bbb aaa bbb Aaa bbb aaa bbb aa bbb  
aaa bbb Aaa bbb aaa bbb Aaa bbb aaa bbb Aaa bbb aaa  
bbb Aaa bbb aaa bbb

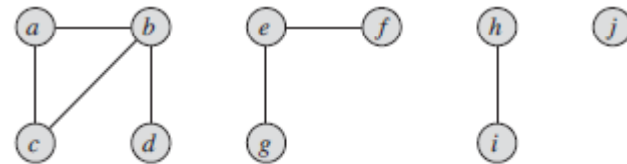
- 教材答疑和讨论
  - TC第21章
  - SB第2章



# 问题1: connected components

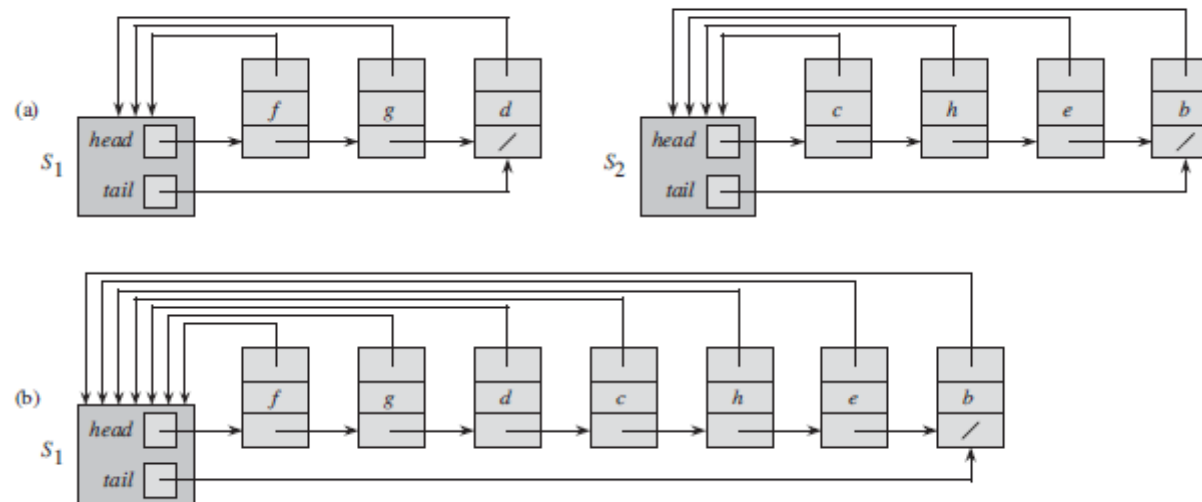
- 你能不能借助这个例子，解释一下Union-Find的三个基本操作的含义？
- 为什么CONNECTED-COMPONENTS的结果是正确的？

```
CONNECTED-COMPONENTS( $G$ )  
1  for each vertex  $v \in G.V$   
2    MAKE-SET( $v$ )  
3  for each edge  $(u, v) \in G.E$   
4    if FIND-SET( $u$ )  $\neq$  FIND-SET( $v$ )  
5      UNION( $u, v$ )
```



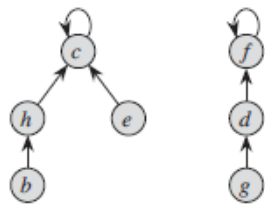
# 问题2: linked-list representation

- 这种实现方式是如何支持Union-Find的三个基本操作的?
- 运行时间分别如何? 瓶颈是哪个操作?
  - 如何利用weighted-union heuristic改进union的运行时间?
- 你能将union的运行时间改进到 $O(1)$ 吗?
  - 付出的代价是什么?



# 问题3: disjoint-set forests

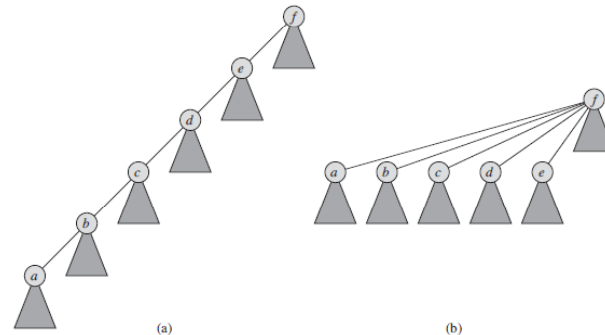
- 这种实现方式是如何支持Union-Find的三个基本操作的?
- 运行时间分别如何? 瓶颈是哪个操作?
  - 如何利用union by rank改进find的运行时间?
  - 如何利用path compression改进find的运行时间?



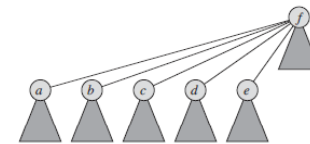
(a)



(b)



(a)



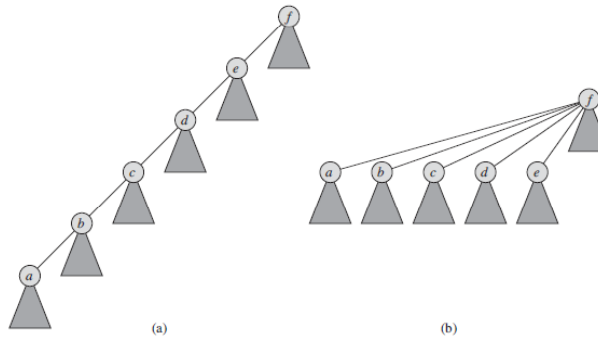
(b)

# 问题3: disjoint-set forests (续)

- FIND-SET想要达到哪些目的?
- 如何证明FIND-SET的正确性?

FIND-SET( $x$ )

```
1 if  $x \neq x.p$   
2    $x.p = \text{FIND-SET}(x.p)$   
3 return  $x.p$ 
```



## 问题4: how many tables

- Today is Ignatius' birthday. He invites a lot of friends. Now it's dinner time. Ignatius wants to know how many tables he needs at least. You have to notice that not all the friends know each other, and all the friends do not want to stay with strangers.

# 问题5: the suspects

- Severe acute respiratory syndrome (SARS), an atypical pneumonia of unknown aetiology, was recognized as a global threat in mid-March 2003. To minimize transmission to others, the best strategy is to separate the suspects from others.
- In the Not-Spreading-Your-Sickness University (NSYSU), there are many student groups. Students in the same group intercommunicate with each other frequently, and a student may join several groups. To prevent the possible transmissions of SARS, the NSYSU collects the member lists of all student groups, and makes the following rule in their standard operation procedure (SOP).
- Once a member in a group is a suspect, all members in the group are suspects.
- However, they find that it is not easy to identify all the suspects when a student is recognized as a suspect. Your job is to write a program which finds all the suspects.

## 问题6: single-linkage agglomerative clustering

- 分组讨论: 如何高效地实现n个元素的single-linkage agglomerative clustering? 需要用到哪些数据结构? 总体的运行时间是多少?
  - 假设元素间距离计算的运行时间:  $O(1)$

