- 作业讲解
  - JH第2章练习2.3.1.7、2.3.1.8、2.3.3.8

JH第2章练习2.3.1.7、2.3.1.8

- 在编码矩阵内容之前，要先编码其规模
  - a11#a12#....#a1n#a21#a22（不知何时换行）
  - 修正：n#a11#a12#... 或a11#a12#...#a1n##a21#...
- 如果没有#，可以用0和1先编码出#
  - 00→0
  - 01→1
  - 11→#

- 教材讨论
  - JH第3章第2、3节

问题1：伪多项式时间算法

- U和Value(h)-U之间有什么联系和区别？
  A是它们中哪个问题的算法？复杂度是多少？

Definition 3.2.1.1. *Let U be an integer-valued problem, and let A be an algorithm that solves U. We say that A is a* **pseudo-polynomial-time algorithm for U** *if there exists a polynomial p of two variables such that*

$$Time_A(x) = O(p(|x|, Max\text{-}Int(x)))$$

for every instance x of U.

Definition 3.2.1.2. *Let U be an integer-valued problem, and let h be a non-decreasing function from $\mathbb{N}$ to $\mathbb{N}$. The* **h-value-bounded subproblem of U, Value(h)-U**, *is the problem obtained from U by restricting the set of all input instances of U to the set of input instances x with $Max\text{-}Int(x) \leq h(|x|)$.*

- 你能给出素数判定问题的一个伪多项式时间算法吗？

问题1：伪多项式时间算法 (续)

- 什么是KP？
  在KP的DP算法中，以下概念各是什么含义？
  - I $I = (w_1, \ldots, w_n, c_1, \ldots, c_n, b)$
  - I_i $I_i = (w_1, w_2, \ldots, w_i, c_1, c_2, \ldots, c_i, b)$
  - triple $(k, W_{i,k}, T_{i,k}) \in \left\{0, 1, 2, \ldots, \sum_{j=1}^{i} c_j\right\} \times \{0, 1, 2, \ldots, b\} \times Pot(\{1, \ldots, i\})$
  - $TRIPLE_i$ $TRIPLE_i$

问题1：伪多项式时间算法 (续)

- 你能基于上述概念解释这个算法的基本过程吗？
- 你能证明这个算法的正确性吗？
  你会计算它的时间复杂度吗？（如何体现伪多项式？）

Algorithm 3.2.2.2 ((DPKP)).

Input: $I = (w_1, w_2, \ldots, w_n, c_1, c_2, \ldots, c_n, b) \in (\mathbb{N} - \{0\})^{2n+1}$, n a positive integer.

Step 1: $TRIPLE(1) := \{(0, 0, \emptyset)\} \cup \{(c_1, w_1, \{1\}) \mid \text{if } w_1 \le b\}$.

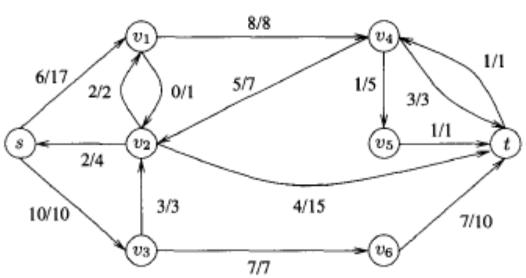
Step 2: **for** $i = 1$ **to** $n - 1$ **do**
  begin $SET(i+1) := TRIPLE(i)$;
    for every $(k, w, T) \in TRIPLE(i)$ **do**
      if $w + w_{i+1} \le b$ **then**
      $SET(i+1) := SET(i+1) \cup \{(k + c_{i+1}, w + w_{i+1}, T \cup \{i+1\})\}$;
    Set $TRIPLE(i+1)$ as a subset of $SET(i+1)$ containing exactly one triple (m, w', T') for every achievable profit m in $SET(i+1)$ by choosing a triple with the minimal weight for the given m
  end

Step 3: Compute $c := \max\{k \in \{1, \ldots, \sum_{i=1}^{n} c_i\} \mid (k, w, T) \in TRIPLE(n)$ for some w and $T\}$.

Output: The index set T such that $(c, w, T) \in TRIPLE(n)$.

问题1：伪多项式时间算法 (续)

- 什么是最大流问题？
- 你能解释Ford-Fulkerson算法的基本过程吗？
- 你会计算它的时间复杂度吗？（如何体现伪多项式？）



Algorithm 3.2.3.10 (The Ford-Fulkerson Algorithm).

Input: $(V, E), c, s, t$ of a network $H = ((V, E), c, \mathbb{Q}^+, s, t)$.

Step 1: Determine an initial flow function f of H (for instance, $f(e) = 0$ for all $e \in E$); $HALT := 0$

Step 2: $S := \{s\}$; $\overline{S} := V - S$;

Step 3: **while** $t \notin S$ and $HALT$=0 **do**
  begin find an edge $e = (u, v) \in E(S, \overline{S}) \cup E(\overline{S}, S)$ such that
    $res(e) > 0$
    –$c(e) - f(e) > 0$ if $e \in E(S, \overline{S})$ and $f(e) > 0$ if $e \in E(\overline{S}, S)$";
    if such an edge does not exist **then** $HALT := 1$
    else if $e \in E(S, \overline{S})$ **then** $S := S \cup \{v\}$
      else $S := S \cup \{u\}$;
    $\overline{S} := V - S$
  end

Step 4: **if** $HALT = 1$ **then return** (f, S)
  else begin find an augmenting path P from s to t, which consists of vertices of S only; –this is possible because both s and t are in S";
    compute $res(P)$;
    determine f' from f as described in Lemma 3.2.3.9
  end;
  goto Step 2

问题2：strongly NP-hard

- 一个strongly NP-hard问题可能存在伪多项式时间算法吗？为什么？

Definition 3.2.4.1. *An integer-valued problem U is called* **strongly NP-hard** *if there exists a polynomial p such that the problem Value(p)-U is* NP-*hard.*

- 如何证明一个问题是strongly NP-hard？
你能以TSP为例来说明吗？
- 为什么这句话成立？
  – Every weighted version of an optimization graph problem (e.g., WEIGHT-VCP) is strongly NP-hard if the original “unweighted” version (e.g., MIN-VCP) is NP-hard.

问题3：参数化

- 从不同的角度，谈谈你对参数化的理解、参数化的意义以及与伪多项式算法的关系

Definition 3.3.1.1. *Let U be a computing problem, and let L be the language of all instances of U. A* **parameterization of U** *is any function* $\boldsymbol{Par}: L \to \mathbb{N}$ *such that*

(i) Par is polynomial-time computable, and
(ii) for infinitely many $k \in \mathbb{N}$, the **k-fixed-parameter set**

$$\boldsymbol{Set_U(k)} = \{x \in L \mid Par(x) = k\}$$

is an infinite set.

We say that A is a **Par-parameterized polynomial-time algorithm for U** *if*

(i) A solves U, and
(ii) there exists a polynomial p and a function $f : \mathbb{N} \to \mathbb{N}$ such that, for every $x \in L$,

$$Time_A(x) \leq f(Par(x)) \cdot p(|x|).$$

问题3：参数化 (续)

- 对于一个问题，可能有多种参数化方法，如何评价其好坏？
  - Capture the inherent difficulty of particular input instances.
  - One can design a practical parameterized polynomial-time algorithm.
  - Most of the problem instances occurring in the considered application have this parameter reasonably small.

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$$Time_A(x) \leq f(Par(x)) \cdot p(|x|).$$

问题3：参数化 (续)

- 什么是VC问题？
- 你理解它的两个参数化算法了吗？
为什么说第二种更好？

Algorithm 3.3.2.4. Input: (G, k), where $G = (V, E)$ is a graph and k is a positive integer.

Step 1: Let H contain all vertices of G with degree greater than k.
if $|H| > k$, **then output**("reject") {Observation 3.3.2.2};
if $|H| \leq k$, **then** $m := k - |H|$ and G' is the subgraph of G obtained
by removing all vertices of H with their incident edges.

Step 2: **if** G' has more than $m(k+1)$ vertices $[|V - H| > m(k+1)]$ **then output**("reject") {Observation 3.3.2.3}.

Step 3: Apply an exhaustive search (by backtracking) for a vertex cover of size at most m in G'.
if G' has a vertex cover of size at most m, **then output**("accept"),
else output("reject").

We consider the following divide-and-conquer strategy. Let (G, k) be an input instance of the vertex cover problem. Take an arbitrary edge $\{v_1, v_2\}$ of G. Let G_i be the subgraph of G obtained by removing v_i with all incident edges from G for $i = 1, 2$. Observe that

$$(G, k) \in \text{VC iff } [(G_1, k-1) \in \text{VC or } (G_2, k-1) \in \text{VC}].$$

Obviously, $(G_i, k-1)$ can be constructed from G in time $O(|V|)$. Since, for every graph H, $(H, 1)$ is a trivial problem that can be decided in $O(|V|)$ time and the recursive reduction of (G, k) to subinstances of (G, k) can result in solving at most 2^k subinstances of (G, k), the complexity of this divide-and-conquer algorithm is in $O(2^k \cdot n)$.