- 教材讨论
  – JH第3章第7节

问题1：用0-1规划建模

- 0-1 KP

$$\text{Maximize} \sum_{i=1}^{n} v_i x_i \text{ subject to } \sum_{i=1}^{n} w_i x_i \leqslant W, \qquad x_i \in \{0,1\}$$

问题1：用0-1规划建模

- multiple KP
  - *n* items and *m* knapsacks with capacities W_i

$$\text{maximize} \sum_{i=1}^{m} \sum_{j=1}^{n} p_j x_{ij}$$

$$\text{subject to} \sum_{j=1}^{n} w_j x_{ij} \le W_i, \text{for all } 1 \le i \le m$$

$$\sum_{i=1}^{m} x_{ij} \le 1, \quad \text{for all } 1 \le j \le n$$

$$x_{ij} \in \{0,1\} \quad \text{for all } 1 \le j \le n \text{ and all } 1 \le i \le m$$

问题1：用0-1规划建模 (续)

- 0-1 multidimensional KP
  - e.g. both a volume limit and a weight limit

$$\text{maximize} \sum_{j=1}^{n} p_j x_j$$

$$\text{subject to} \sum_{j=1}^{n} w_{ij} x_j \leq W_i, \quad \text{for all } 1 \leq i \leq m$$

$$x_j \in \{0,1\}$$

问题1：用0-1规划建模 (续)

- SCP

$$\text{minimize} \sum_{i=1}^{m} x_i$$

under the following n linear constraints

$$\sum_{j \in \text{Index}(k)} x_j \geq 1 \quad \text{for } k = 1, \ldots, n.$$

问题1：用0-1规划建模 (续)

- MS

$$
\begin{aligned}
\text{minimize} \quad & t \\
\text{subject to} \quad & \sum_{i \in M} x_{ij} = 1, && j \in J \\
& \sum_{j \in J} x_{ij} p_{ij} \le t, && i \in M \\
& x_{ij} \in \{0, 1\}, && i \in M,\ j \in J
\end{aligned}
$$

问题1：用0-1规划建模 (续)

- MAX-SAT

$$
\begin{aligned}
\text{maximize} \quad & \sum_{c \in \mathcal{C}} z_c \\
\text{subject to} \quad & \forall c \in \mathcal{C}: \ \sum_{i \in S_c^+} y_i + \sum_{i \in S_c^-} (1 - y_i) \geq z_c \\
& \forall c \in \mathcal{C}: \ z_c \in \{0, 1\} \\
& \forall i: \ y_i \in \{0, 1\}
\end{aligned}
$$

问题1：用0-1规划建模 (续)

- The **facility location problem** consists of a set of potential facility *F* that can be opened, and a set of cities *C* that must be serviced. The goal is to pick a subset of facilities to open, to minimize the sum of distances from each city to its nearest facility, plus the sum of opening costs of the facilities.

$$
\begin{array}{lll}
\text{minimize} & \displaystyle\sum_{i\in F,\, j\in C} c_{ij}x_{ij} + \sum_{i\in F} f_i y_i & \\
\text{subject to} & \displaystyle\sum_{i\in F} x_{ij} \geq 1, & j \in C \\
& y_i - x_{ij} \geq 0, & i \in F,\ j \in C \\
& x_{ij} \in \{0,1\}, & i \in F,\ j \in C \\
& y_i \in \{0,1\}, & i \in F
\end{array}
$$

问题1：用0-1规划建模 (续)

- WEIGHT-VCP

minimize

$$\sum_{i=1}^{n} c(v_i) \cdot x_i.$$

$x_i \in \{0, 1\}$

$x_i + x_j \geq 1$ for every $\{v_i, v_j\} \in E$

问题1：用0-1规划建模 (续)

- maximum matching

maximize

$$\sum_{e \in E} x_e$$

under the $|V|$ constraints

$$\sum_{e \in E(v)} x_e \leq 1 \quad \text{for every } v \in V,$$

and the following $|E|$ constraints

$$x_e \in \{0,1\} \quad \text{for every } e \in E.$$

问题1：用0-1规划建模 (续)

- WEIGHT-CL

$$
\begin{aligned}
&\max \sum_{i=1}^{n} w_i x_i, \\
&\text{s.t. } x_i + x_j \le 1, \ \forall\, (i,j) \in \overline{E}, \\
&x_i \in \{0,1\}, \ i = 1, \ldots, n.
\end{aligned}
$$

问题1：用0-1规划建模 (续)

- TSP

$$\min z = \sum_{j=2}^{j=n} \sum_{i=1}^{j-1} c_{ij} x_{ij},$$

subject to: $x_{ij} = 0, 1, \quad (i = 1, \cdots, j-1; j = 2, \cdots, n)$

and the loop constraints

$$\sum_{i \in S} \sum_{j \in \bar{S}} x_{ij} \geqq 2,$$

for all nonempty partitions $(S, \bar{S})$ such that if $(S, \bar{S})$ is considered $(\bar{S}, S)$ is not.

问题2：rounding

- 什么是一个好的rounding？
  - The obtained rounded integral solution is a feasible solution.
  - The cost has not been changed too much.

问题2：rounding (续)

- SCP(k)可以怎样rounding？得到的结果有多好？

$$\text{minimize} \sum_{i=1}^{m} x_i$$

under the constraints

$$\sum_{h \in Index(a_j)} x_h \geq 1 \text{ for } j = 1, \ldots, n,$$

$$x_i \in \{0, 1\} \text{ for } i = 1, \ldots, m,$$

问题2：rounding (续)

- WEIGHT-VCP可以怎样rounding？得到的结果有多好？

minimize

$$\sum_{i=1}^{n} c(v_i) \cdot x_i.$$

$x_i \in \{0, 1\}$

$x_i + x_j \geq 1$ for every $\{v_i, v_j\} \in E$

问题2： rounding (续)

- 如果我们增加一个约束呢？

minimize

$$\sum_{i=1}^{n} c(v_i) \cdot x_i.$$

$x_i \in \{0, 1\}$

$x_i + x_j \geq 1$ for every $\{v_i, v_j\} \in E$

$$\sum_{i=1}^{n} x_i \geq k$$

- iterative rounding

问题2： rounding (续)

- MAX-SAT可以怎样rounding？你会分析结果的好坏吗？

$$
\begin{aligned}
\text{maximize} \quad & \sum_{c \in \mathcal{C}} z_c \\
\text{subject to} \quad & \forall c \in \mathcal{C}: \sum_{i \in S_c^+} y_i + \sum_{i \in S_c^-} (1 - y_i) \geq z_c \\
& \forall c \in \mathcal{C}: \ z_c \in \{0, 1\} \\
& \forall i: \ y_i \in \{0, 1\}
\end{aligned}
$$

- randomized rounding

问题2：rounding (续)

- 你能用类似的方法处理WEIGHT-VCP吗？发现新问题了吗？

minimize

$$\sum_{i=1}^{n} c(v_i) \cdot x_i.$$

$x_i \in \{0, 1\}$

$x_i + x_j \geq 1$ for every $\{v_i, v_j\} \in E$

问题3：广义的relaxation

- relaxation并不限于LP，它是一种思想
  - 在更大范围内求解更容易的一个问题，再修正到原始范围
- 你能利用这个思想给出一个求最长哈密尔顿圈的近似算法吗？
  - 提示：哈密尔顿圈和匹配之间有什么关系？
- 偶数个顶点：哈密尔顿圈=2个匹配
  - 将找最长哈密尔顿圈松弛为找一个最大匹配（再补齐为哈密尔顿圈）
  - 得到的结果有多好？
- 奇数个顶点：哈密尔顿圈=3个匹配
  - 同理