3-4 Graph

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CZ-1.2

Create an example of your own similar to Example 1.1 with nine editors and eight committees and then draw the corresponding graph.

Example 1.1 The ten editors have decided on the seven committees: $c_1 = \{1, 2, 3\}$, $c_2 = \{1, 3, 4, 5\}$, $c_3 = \{2, 5, 6, 7\}$, $c_4 = \{4, 7, 8, 9\}$, $c_5 = \{2, 6, 7\}$, $c_6 = \{8, 9, 10\}$, $c_7 = \{1, 3, 9, 10\}$. They have set aside three time periods for the seven committees to meet on those Fridays when all ten editors are present. Some pairs of committees cannot meet during the same period because one or two of the editors are on both committees. This situation can be modeled visually as shown in Figure 1.1.

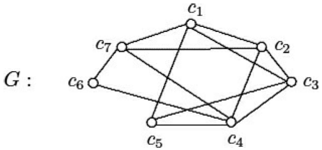
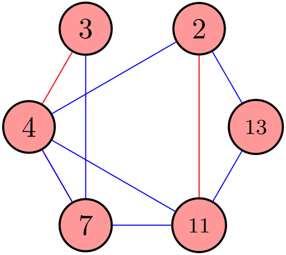


Figure 1.1: A graph

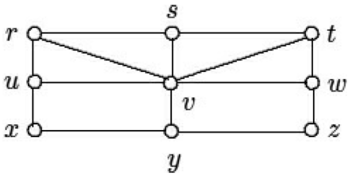
CZ-1.3

Let $S = \{2, 3, 4, 7, 11, 13\}$. Draw the graph G whose vertex set is S and such that $ij \in E(G)$ for $i, j \in S$ if $i + j \in S$ or $|i - j| \in S$.

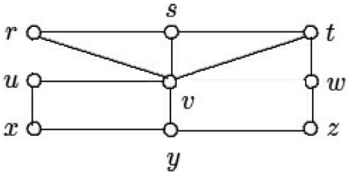


CZ-1.11

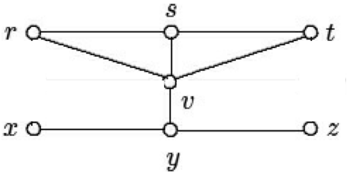
Let G be the graph of Figure 1.20, let $X = \{e, f\}$, where $e = ru$ and $f = vw$, and let $U = \{u, w\}$. Draw the subgraphs $G - X$ and $G - U$ of G.



$G - X$

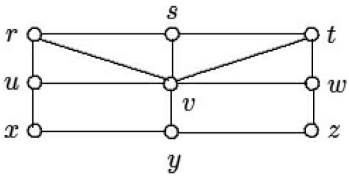


$G - U$



CZ-1.12

For the graph G of Figure 1.20, give an example of each of the following or explain why no such example exists.



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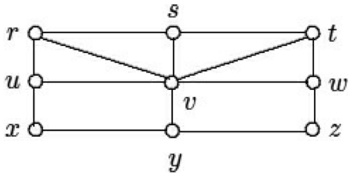
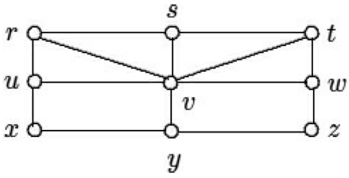


(a) An $x - y$ walk of length 6.

CZ-1.12

For the graph G of Figure 1.20, give an example of each of the following or explain why no such example exists.



(a) An $x - y$ walk of length 6.

$x \to u \to r \to v \to r \to v \to y$

CZ-1.12

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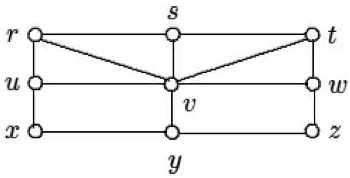

(b) A $v - w$ **trail** that is not a $v - w$ path.

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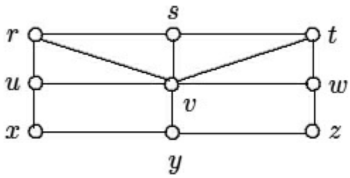

(b) A $v-w$ **trail** that is not a $v-w$ path.

Trail: a walk with no repeated edges

$v \to r \to s \to v \to w$

CZ-1.12

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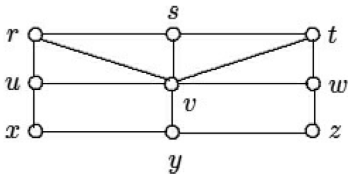

(c) An $r - z$ path of length 2.

CZ-1.12

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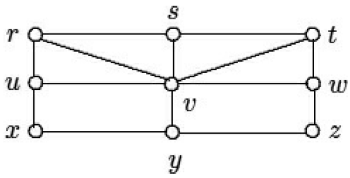


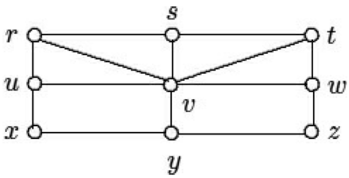
(c) An $r - z$ path of length 2.

path: a walk with no repeated vertices.

Not exists!

CZ-1.12

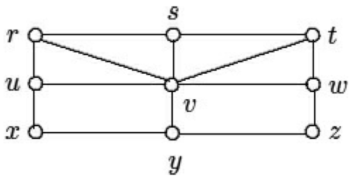
For the graph G of Figure 1.20, give an example of each of the following or explain why no such example exists.



(d) An $x - z$ path of length 3.

CZ-1.12

For the graph G of Figure 1.20, give an example of each of the following or explain why no such example exists.



(d) An $x - z$ path of length 3.

Not exists!

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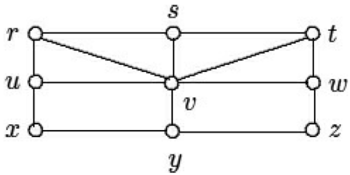
For the graph G of Figure 1.20, give an example of each of the following or explain why no such example exists.



(e) An $x - t$ path of length $d(x, t)$.

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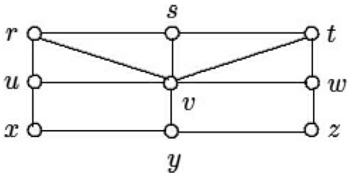

(e) An $x - t$ path of length $d(x, t)$.

$d(x, t) = 3$

$x \to y \to v \to t$

CZ-1.12

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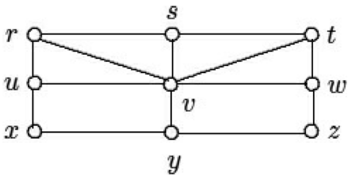

(f) A **circuit** of length 10.

CZ-1.12

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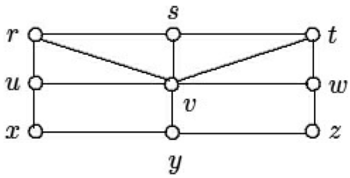

(f) A **circuit** of length 10.

A **circuit** in a graph G is a closed trail of length 3 or more.

$x \to y \to z \to w \to v \to t \to s \to v \to r \to u \to x$

CZ-1.12

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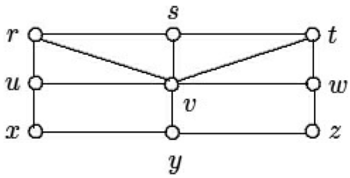

(g) A cycle of length 8.

CZ-1.12

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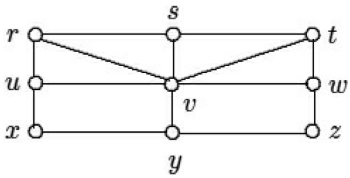

(g) A cycle of length 8.

Cycle: A circuit that repeats no vertex.

$x \to y \to z \to w \to t \to s \to r \to u \to x$

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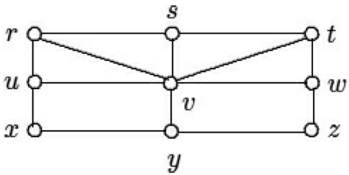

(h) A geodesic whose length is $diam(G)$.

CZ-1.12

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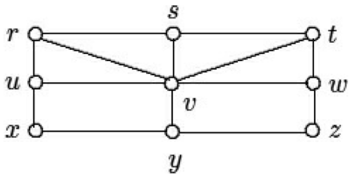

(h) A geodesic whose length is $diam(G)$.

$diam(G)$: The **greatest** distance between **any** two vertices of a connected graph G

$$diam(G) = 3$$

$$x \to y \to v \to t$$

CZ-2.1

Give an example of the following or explain why no such example exists:

(1) a graph of order 7 whose vertices have degrees 1, 1, 1, 2, 2, 3, 3.

(2) a graph of order 7 whose vertices have degrees 1, 2, 2, 2, 3, 3, 7.

(3) a graph of order 4 whose vertices have degrees 1, 3, 3, 3.

CZ-2.1

Give an example of the following or explain why no such example exists:

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(3) a graph of order 4 whose vertices have degrees 1, 3, 3, 3.

(1) No!

CZ-2.1

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(1) No! $\sum_{v \in V(G)} deg(v)$ is odd

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(1) No! $\sum_{v \in V(G)} deg(v)$ is odd

(2) No!

CZ-2.1

Give an example of the following or explain why no such example exists:

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(1) No! $\sum_{v \in V(G)} deg(v)$ is odd

(2) No! $\Delta(G) = n$

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(3) a graph of order 4 whose vertices have degrees 1, 3, 3, 3.

(1) No! $\sum_{v \in V(G)} deg(v)$ is odd

(2) No! $\Delta(G) = n$

(3) No! $\sum_{v \in V(G)} deg(v) = 10 > \frac{4 \times 3}{2} = 6$

CZ-2.19

Construct an r-regular graph of order 6 and an s-regular graph of order 7 for all possible values of r and s.

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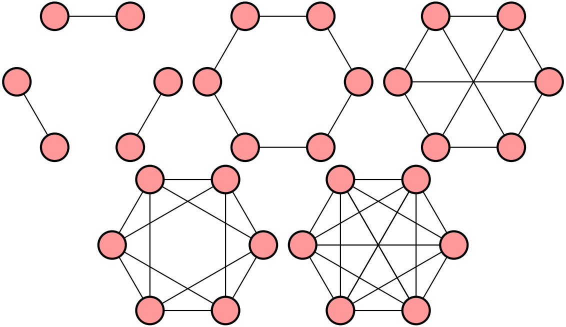
Order 6:



CZ-2.19

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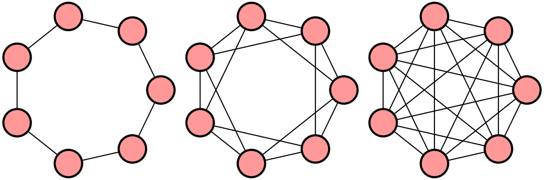
CZ-2.19

Construct an r-regular graph of order 6 and an s-regular graph of order 7 for all possible values of r and s.

Order 7:



CZ 2.31

Prove that a sequence $d_1, d_2, \cdots, d_n$ is graphical **if and only if** $n - d_1 - 1, n - d_2 - 1, \cdots, n - d_n - 1$ is graphical.

CZ 2.31

Prove that a sequence $d_1, d_2, \cdots, d_n$ is graphical **if and only if** $n - d_1 - 1, n - d_2 - 1, \cdots, n - d_n - 1$ is graphical.

- Let K_n be the complete graph of order n
- If a sequence $d_1, d_2, \cdots, d_n$ is graphical, let G_n be such a satisfactory graph
- Then, $\bar{G_n} = K_n - G_n$ is the complement graph of G_n satisfying the degree sequence: $n - d_1 - 1, n - d_2 - 1, \cdots, n - d_n - 1$

CZ 3.1

Give an example of three different (non-isomorphic) graphs of order 5 and size 5.

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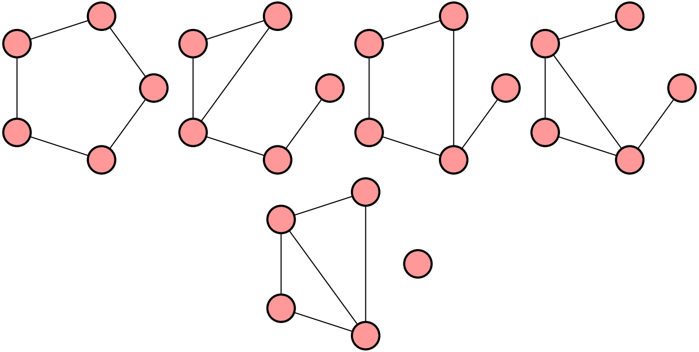
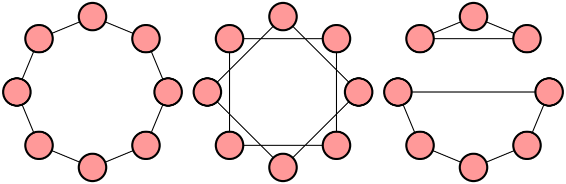


CZ 3.2

Give an example of three graphs of the same order, same size and same degree sequence such that no two of these graphs are isomorphic.

CZ 3.2

Give an example of three graphs of the same order, same size and same degree sequence such that no two of these graphs are isomorphic.



Thank You!