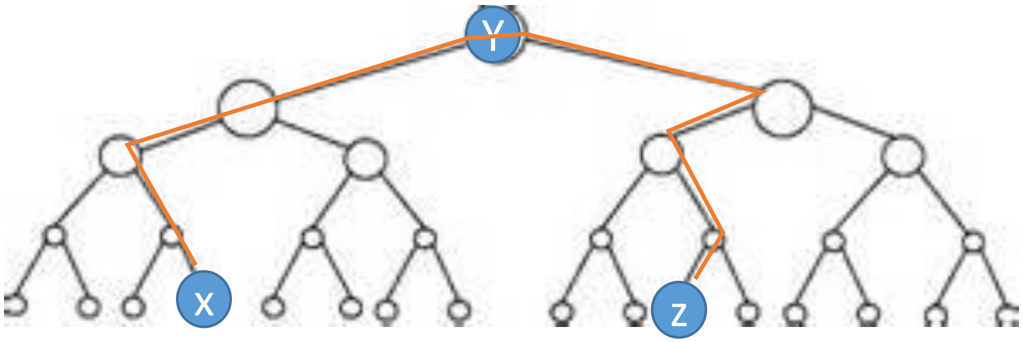
习题2-13(1)

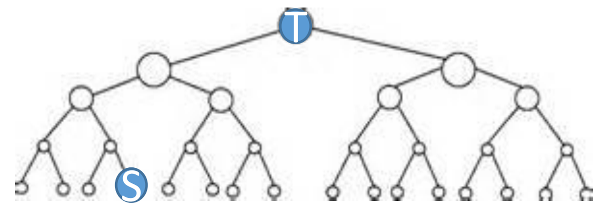
12.2-8

Prove that no matter what node we start at in a height-h binary search tree, k successive calls to TREE-SUCCESSOR take $O(k + h)$ time.

- Let x be the starting node and z be the ending node after k successive calls to TREE-SUCCESSOR().
- Let p be the simple path between x and z inclusive.
- Let y be the common ancestor of x and z that p visits.
- The length of p is at most 2h, which is O(h).
- Let output be the elements that their values are between x.key and z.key inclusive.
- The size of output is O(k).
- In the execution of k successive calls to TREE-SUCCESSOR, the nodes that are in p are visited, and besides the nodes x, y and z, if a sub tree of a node in p is visited then all its elements are in output.
- So the running time is O(h+k).

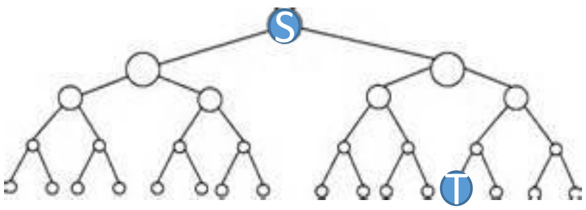


证明2（分情况）



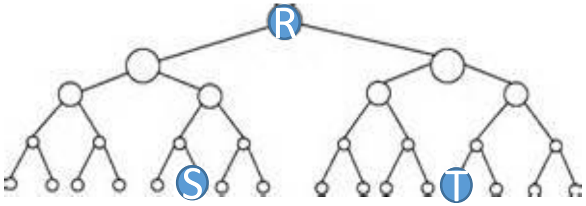
- S，T分别表示搜索起点、终点
- CASE1：S在T的（左）子树中
  - 从S➔T的路径（ $a_1 \rightarrow a_2 \rightarrow \cdots \rightarrow a_m$ ，$a_1 = S, a_m = T$ ）中的每个节点$a_i (i < m)$, 若a_{i-1}为其左儿子，则a_i及其右子树中所有节点的值必然介于S，T之间
  - 从S到T的搜索过程遍历了上述路径中的所有节点，以及满足条件的所有右子树
  - 路径长度O(h)
  - 所有右子树中节点数<=k, O(K)
  - 总数:O(h)+O(k)=O(h+k)

证明2（分情况）



- S，T分别表示搜索起点、终点
- CASE2：T在S的（又）子树中
  - 从S➔T的路径（ $a_1 \rightarrow a_2 \rightarrow \cdots \rightarrow a_m$ ，$a_1 = S, a_m = T$ ）中的每个节点$a_i (i < m)$,若a_{i+1}为其又儿子，则a_i及其左子树中所有节点的值必然介于S，T之间
  - 从S到T的搜索过程遍历了上述路径中的所有节点，以及满足条件的所有左子树
  - 路径长度O(h)
  - 所有左子树中节点数<=k, O(K)
  - 总数:O(h)+O(k)=O(h+k)

证明2（分情况）



- S，T分别表示搜索起点、终点
- CASE3：T，S分属不同子树，其最小公共root为R
  - 分为两段：
    - S→R(包含k_1个S,T之间数):CASE1
      - $O(h+k_1)$
    - R→T(包含k_1个S,T之间数):CASE2
      - $O(h+k_2)$
  - 汇总： $O(h+k_1)+O(h+k_2)=O(h+k)$