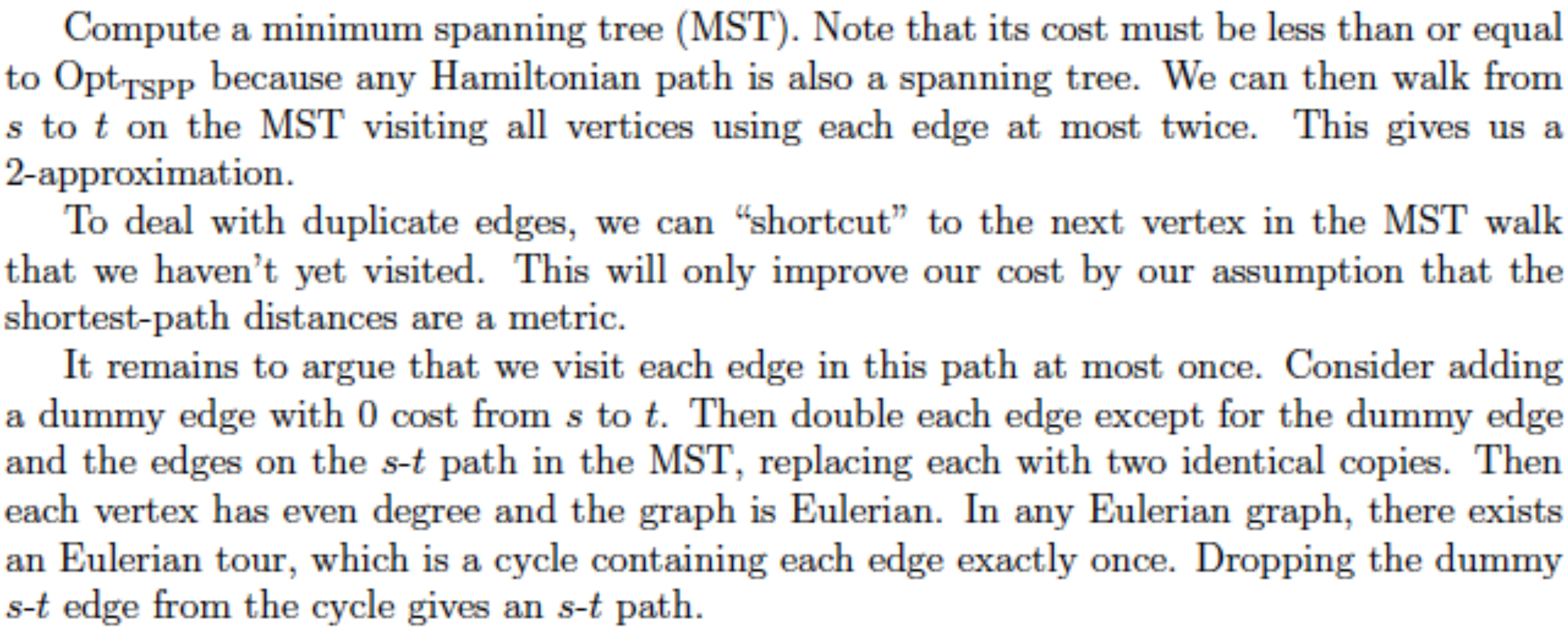
- 作业讲解
  - 设计一个找最小完美匹配的算法
  - JH第4章练习4.3.5.6、4.3.5.11、4.3.5.13

设计一个找最小完美匹配的算法

- 基于Edmonds Blossom Algorithm的primal-dual

JH第4章练习4.3.5.6

- 2-approximation



Compute a minimum spanning tree (MST). Note that its cost must be less than or equal to $\mathrm{Opt}_{\mathrm{TSPP}}$ because any Hamiltonian path is also a spanning tree. We can then walk from s to t on the MST visiting all vertices using each edge at most twice. This gives us a 2-approximation.

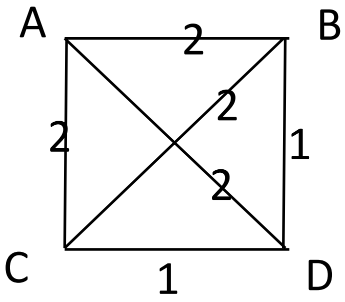
To deal with duplicate edges, we can "shortcut" to the next vertex in the MST walk that we haven't yet visited. This will only improve our cost by our assumption that the shortest-path distances are a metric.

It remains to argue that we visit each edge in this path at most once. Consider adding a dummy edge with 0 cost from s to t. Then double each edge except for the dummy edge and the edges on the s-t path in the MST, replacing each with two identical copies. Then each vertex has even degree and the graph is Eulerian. In any Eulerian graph, there exists an Eulerian tour, which is a cycle containing each edge exactly once. Dropping the dummy s-t edge from the cycle gives an s-t path.

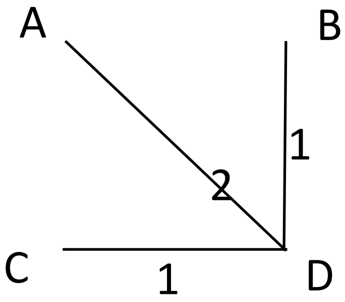
JH第4章练习4.3.5.6

- 5/3-approximation
  - J.A. Hoogeveen.
    Analysis of Christofides' heuristic: Some paths are more difficult than cycles.
    Operations Research Letters 10 (1991) 291-295.

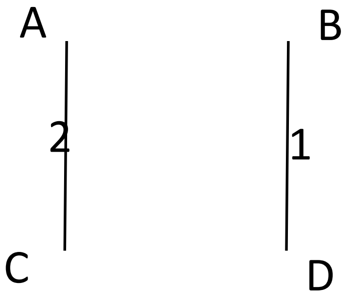
JH第4章练习4.3.5.11



G



MST



最小权完美匹配

算法4.3.5.1：BDCAB=6
Christofides：BDACB=7

- 教材讨论
  - JH第5章第1、2节

问题1：随机算法的基本概念

- 你能从图灵机的角度分别阐述对于随机算法的这些理解吗？
  - a nondeterministic algorithm that has a probability distribution for every nondeterministic choice
  - a deterministic algorithm with an additional input that consists of a sequence of random bits
  - a set of deterministic algorithms from which one algorithm is randomly chosen for the given input

- 你能从上述这些角度分别解释Random$_A$(x)吗？
- 如果Random$_A$(x)不超过对数，意味着什么？

问题1：随机算法的基本概念 (续)

- 你能解释这两种时间复杂度的计算方式吗？
- 它们分别存在什么问题？怎么解决？

$$\boldsymbol{Exp\text{-}Time_A(x)} = E[\mathit{Time}] = \sum_{C} \mathit{Prob}_{A,x}(C) \cdot \mathit{Time}(C)$$

$$\boldsymbol{Exp\text{-}Time_A(n)} = \max\{\mathit{Exp\text{-}Time}_A(x) \,|\, x \text{ is an input of size } n\}$$

$$\boldsymbol{Time_A(x)} = \max\{\mathit{Time}(C) \,|\, C \text{ is a run of } A \text{ on } x\}$$

$$\boldsymbol{Time_A(n)} = \max\{\mathit{Time}_A(x) \,|\, x \text{ is an input of size } n\}$$

问题2：Las Vegas算法

- Las Vegas和Monte Carlo算法的区别是什么？

- 你理解Las Vegas算法的两种定义了吗？

$$Prob(A(x) = F(x)) = 1$$

$$Prob(A(x) = F(x)) \geq \frac{1}{2}$$

$$Prob(A(x) = \text{“?”}) = 1 - Prob(A(x) = F(x)) \leq \frac{1}{2}$$

- 它们分别采用了哪种时间复杂度的计算方式？
- 为什么会有这种区别？

问题2：Las Vegas算法 (续)

- 你能画个图解释一下one-way communication protocol吗？

- Choice$_n$是F的一个例子，它的直观含义是什么？
- 针对这个例子，P349的Las Vegas算法的思路是什么？
- 和一般的确定性算法相比，该算法优劣分别是什么？
- 该算法符合Las Vegas算法两种定义中的哪一种？

- 你能不能改造这个算法，使它符合另一种定义？
- 改造之后，上述优劣发生了怎样的变化？

问题3：Monte Carlo算法

- 你能解释one/two-sided-error Monte Carlo算法吗？

  (i) for every $x \in L$, $Prob(A(x) = 1) \geq 1/2$, and

  (ii) for every $x \notin L$, $Prob(A(x) = 0) = 1$.

  $$Prob(A(x) = F(x)) \geq \frac{1}{2} + \varepsilon.$$

- 它们在具体应用中分别如何使用？
- 我们为什么没有讨论它们的时间复杂度？转而讨论了什么？

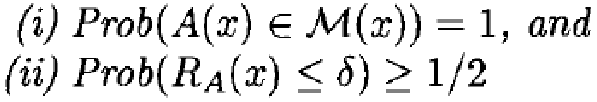
- unbounded-和two-sided error Monte Carlo算法的区别是什么？

  $$Prob(A(x) = F(x)) > \frac{1}{2}.$$

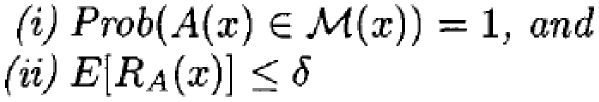
- 这种区别造成了什么结果？

问题4：随机优化算法

- 你理解randomized δ-approximation 和randomized δ-expected approximation算法了吗？



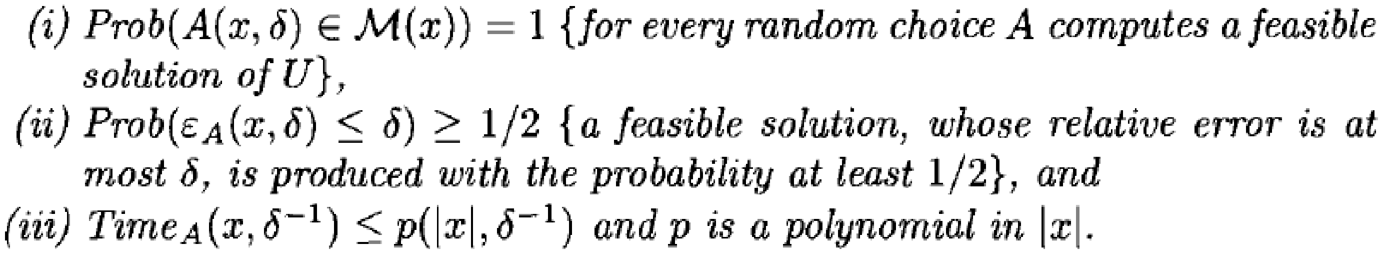
(i) $Prob(A(x) \in \mathcal{M}(x)) = 1$, *and*
(ii) $Prob(R_A(x) \leq \delta) \geq 1/2$



(i) $Prob(A(x) \in \mathcal{M}(x)) = 1$, *and*
(ii) $E[R_A(x)] \leq \delta$

- 这两种算法之间有什么关系？

- 你理解RPTAS了吗？



(i) $Prob(A(x, \delta) \in \mathcal{M}(x)) = 1$ {*for every random choice A computes a feasible solution of U*},
(ii) $Prob(\varepsilon_A(x, \delta) \leq \delta) \geq 1/2$ {*a feasible solution, whose relative error is at most* δ, *is produced with the probability at least* $1/2$}, *and*
(iii) $Time_A(x, \delta^{-1}) \leq p(|x|, \delta^{-1})$ *and* p *is a polynomial in* $|x|$.

- 它和PTAS的区别是什么？

问题5：随机算法的设计范式

- 你理解这三类范式的思想了吗？能不能各举一个例子？
  - Foiling an adversary
  - Abundance of witnesses (& fingerprinting)
  - Random sampling (& relaxation and random rounding)