

- 教材讨论
  - TJ第12、13、14章

# 问题1: general linear group

- 你能从矩阵和线性变换两个角度来解释 general linear group 吗？  
（群中的元素、运算分别是什么？）
  - all  $n \times n$  invertible matrices, matrix multiplication
  - invertible linear transformations, composition
- 它为什么是一个群？

# 问题2: special linear group

- 什么是special linear group?
  - general linear group & det=1
- 它为什么是一个群?
- 它在二维空间上的几何意义是什么? 为什么?
  - 保持面积不变
  - The area of the parallelogram spanned by  $\mathbf{a}$  and  $\mathbf{b}$  is  $\left| \det \left( \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \right) \right|$ .
- 请你构造一个determinant=-1的矩阵, 试试看它是不是也能保持面积不变
- 你觉得determinant=1和determinant=-1在几何意义上有什么区别?
  - “方向”不同
- 你能不能在更简单的一维空间上解释这一区别?
- 在更复杂的三维空间上呢?

[http://mathinsight.org/determinant\\_linear\\_transformation](http://mathinsight.org/determinant_linear_transformation)

# 问题3: orthogonal group和isometry

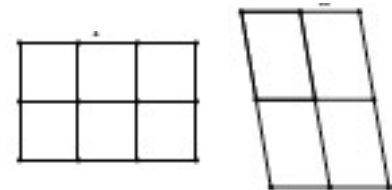
- 什么是orthogonal group?
  - general linear group &  $A^{-1}=A^t$
- 它为什么是一个群?
- 它的几何意义是什么?
  - 保持距离/长度/内积不变...
- isometry group的几何意义是什么?
  - 保持距离不变
- 那么orthogonal group和isometry group有什么区别?
  - 是否保持原点不变
- 在几何意义上, 你能举一个属于isometry group但不属于orthogonal group的变换吗?
  - translation

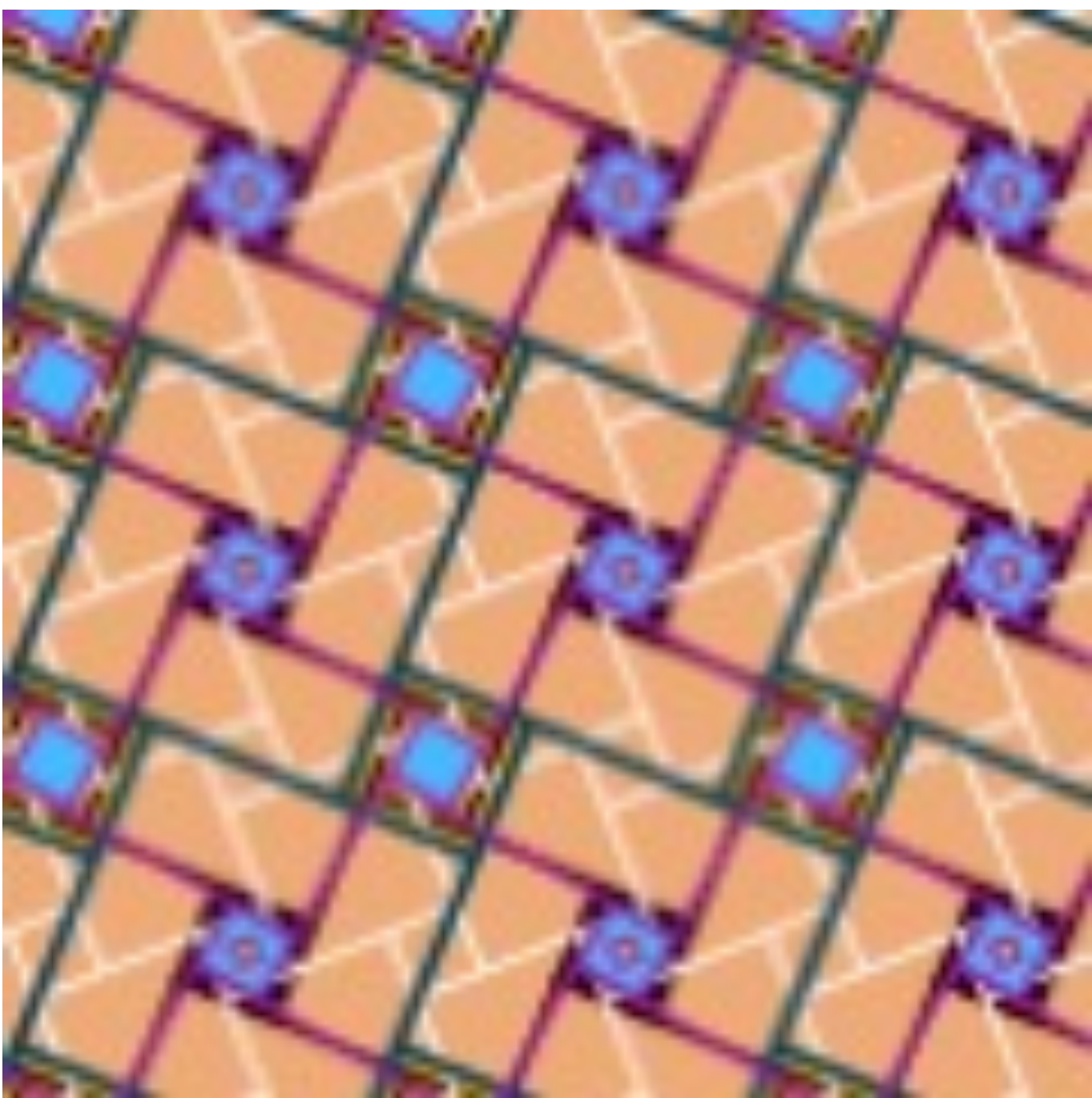
# 问题3: orthogonal group和isometry (续)

- 因此, 在几何意义上, orthogonal group中都是一些什么样的线性变换?
  - rotation (围绕原点)
  - reflection (对称轴过原点/原点对称)
- orthogonal group中的矩阵的行列式有什么特征?
  - determinant= $\pm 1$
- 因此, 在几何意义上, orthogonal group和special linear group的交集special orthogonal group中只剩下哪些线性变换?
  - rotation (围绕原点)
- 顺便问一下, 你能发现rotation和reflection之间的关系吗?
  - rotation = 两次reflection
- 现在你能抛开rotation, 只用reflection来解释orthogonal group和special orthogonal group吗?
  - 若干次reflection生成orthogonal group
  - 偶数次reflection生成special orthogonal group

# 问题4: symmetry group和wallpaper group

- 什么是symmetry group?
  - isometry group & some points fixed
- 刚才提到的这些群中, 哪些是symmetry group?
  - general linear group
  - special linear group
  - orthogonal group
  - special orthogonal group
- 什么是wallpaper group? 它和symmetry group有什么关系?
  - symmetry group的广义定义: some **objects** (e.g. points, **patterns**) fixed
- 如何理解“两张wallpaper对应同一个wallpaper group”?



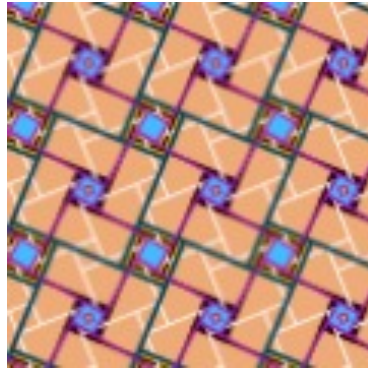


## 问题4: symmetry group和wallpaper group (续)

- 以下这些wallpaper对应同一个wallpaper group吗?

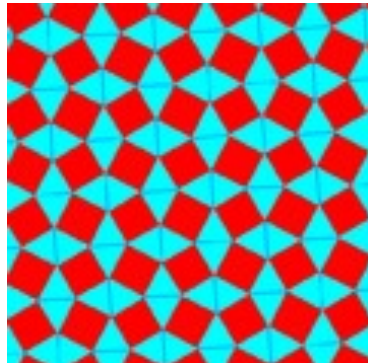
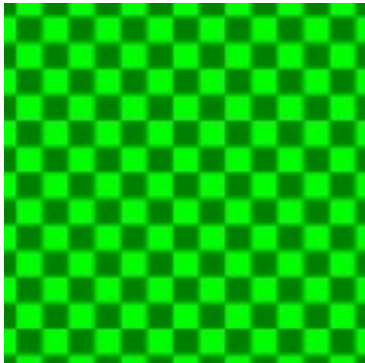


(不考虑颜色)



p4: 2种90度转点, 1种180度转点

现在你理解为什么说p4的point group与 $Z_4$ 同构了吗?



p4m: 四向对称轴; 90度转点在对称轴上

p4g: 两向对称轴; 90度转点不在对称轴上



## 问题4: symmetry group和wallpaper group (续)

- 你能为p1—p6各画出一种wallpaper吗?

Notation and Space Groups	Point Group	Lattice Type	Reflections or Glide Reflections?
p1	$Z_1$	parallelogram	none
p2	$Z_2$	parallelogram	none
p3	$Z_3$	hexagonal	none
p4	$Z_4$	square	none
p6	$Z_6$	hexagonal	none
pm	$D_1$	rectangular	reflections
pg	$D_1$	rectangular	glide reflections
cm	$D_1$	rhombic	both
pmm	$D_2$	rectangular	reflections
pmg	$D_2$	rectangular	glide reflections
pgg	$D_2$	rectangular	both
c2mm	$D_2$	rhombic	both
p3m1, p31m	$D_3$	hexagonal	both
p4m, p4g	$D_4$	square	both
p6m	$D_6$	hexagonal	both

# 问题4: symmetry group和wallpaper group (续)

- p1

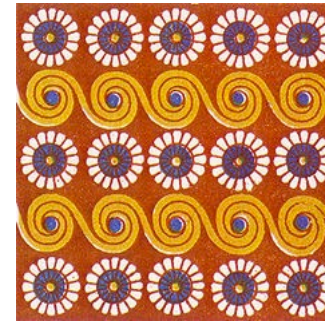
Notation and Space Groups	Point Group	Lattice Type	Reflections or Glide Reflections?
p1	$Z_1$	parallelogram	none
p2	$Z_2$	parallelogram	none
p3	$Z_3$	hexagonal	none
p4	$Z_4$	square	none
p6	$Z_6$	hexagonal	none
pm	$D_1$	rectangular	reflections
pg	$D_1$	rectangular	glide reflections
cm	$D_1$	rhombic	both
pmm	$D_2$	rectangular	reflections
pmg	$D_2$	rectangular	glide reflections
pgg	$D_2$	rectangular	both
c2mm	$D_2$	rhombic	both
p3m1, p31m	$D_3$	hexagonal	both
p4m, p4g	$D_4$	square	both
p6m	$D_6$	hexagonal	both



# 问题4: symmetry group和wallpaper group (续)

- p2

Notation and Space Groups	Point Group	Lattice Type	Reflections or Glide Reflections?
p1	$Z_1$	parallelogram	none
p2	$Z_2$	parallelogram	none
p3	$Z_3$	hexagonal	none
p4	$Z_4$	square	none
p6	$Z_6$	hexagonal	none
pm	$D_1$	rectangular	reflections
pg	$D_1$	rectangular	glide reflections
cm	$D_1$	rhombic	both
pmm	$D_2$	rectangular	reflections
pmg	$D_2$	rectangular	glide reflections
pgg	$D_2$	rectangular	both
c2mm	$D_2$	rhombic	both
p3m1, p31m	$D_3$	hexagonal	both
p4m, p4g	$D_4$	square	both
p6m	$D_6$	hexagonal	both



# 问题4: symmetry group和wallpaper group (续)

- p3

Notation and Space Groups	Point Group	Lattice Type	Reflections or Glide Reflections?
p1	$Z_1$	parallelogram	none
p2	$Z_2$	parallelogram	none
p3	$Z_3$	hexagonal	none
p4	$Z_4$	square	none
p6	$Z_6$	hexagonal	none
pm	$D_1$	rectangular	reflections
pg	$D_1$	rectangular	glide reflections
cm	$D_1$	rhombic	both
pmm	$D_2$	rectangular	reflections
pmg	$D_2$	rectangular	glide reflections
pgg	$D_2$	rectangular	both
c2mm	$D_2$	rhombic	both
p3m1, p31m	$D_3$	hexagonal	both
p4m, p4g	$D_4$	square	both
p6m	$D_6$	hexagonal	both



# 问题4: symmetry group和wallpaper group (续)

- p4

Notation and Space Groups	Point Group	Lattice Type	Reflections or Glide Reflections?
p1	$Z_1$	parallelogram	none
p2	$Z_2$	parallelogram	none
p3	$Z_3$	hexagonal	none
p4	$Z_4$	square	none
p6	$Z_6$	hexagonal	none
pm	$D_1$	rectangular	reflections
pg	$D_1$	rectangular	glide reflections
cm	$D_1$	rhombic	both
pmm	$D_2$	rectangular	reflections
pmg	$D_2$	rectangular	glide reflections
pgg	$D_2$	rectangular	both
c2mm	$D_2$	rhombic	both
p3m1, p31m	$D_3$	hexagonal	both
p4m, p4g	$D_4$	square	both
p6m	$D_6$	hexagonal	both



# 问题4: symmetry group和wallpaper group (续)

- p6

Notation and Space Groups	Point Group	Lattice Type	Reflections or Glide Reflections?
p1	$Z_1$	parallelogram	none
p2	$Z_2$	parallelogram	none
p3	$Z_3$	hexagonal	none
p4	$Z_4$	square	none
p6	$Z_6$	hexagonal	none
pm	$D_1$	rectangular	reflections
pg	$D_1$	rectangular	glide reflections
cm	$D_1$	rhombic	both
pmm	$D_2$	rectangular	reflections
pmg	$D_2$	rectangular	glide reflections
pgg	$D_2$	rectangular	both
c2mm	$D_2$	rhombic	both
p3m1, p31m	$D_3$	hexagonal	both
p4m, p4g	$D_4$	square	both
p6m	$D_6$	hexagonal	both

