平面图与图着色
可平面图(Planar Graph)

• 如果图G能够被画在一个平面上且图中的任意两条边都不相交，则图G被称为可平面图。
Regions

- Exterior region
- Boundary of region
The Euler Identity

• Theorem 9.1
  – If $G$ is a connected plane graph of order $n$, size $m$ and having $r$ regions, then $n-m+r =2$. 
Theorem 9.2

• If $G$ is a planar graph of order $n \geq 3$ and size $m$, then $m \leq 3n-6$.

• Corollary 9.3
  – Every planar graph contains a vertex of degree 5 or less.

• Corollary 9.4
  – $K_5$ is nonplanar.
Theorem 9.5

• The graph $K_{3,3}$ is nonplanar.
Kuratowski’s theorem

• A graph $G$ is planar if and only if $G$ does not contain $K_5$, $K_{3,3}$, or a subdivision of $K_5$ or $K_{3,3}$ as a subgraph.

  – A graph $G’$ is called a subdivision of a graph $G$ if one or more vertices of degree 2 are inserted into one or more edges of $G$. 
Graph Coloring

- Dated back to 1852, Francis Guthrie
- → De Morgan →
Vertex Coloring

• Assignment of colors to the vertices of G, one color to each vertex, such that adjacent vertices are colored differently.
• Chromatic number, $\chi (G)$
• k-colorable; k-coloring; k-chromatic.
The Four Color Theorem

• The chromatic number of every planar graph is at most 4.
Theorem 10.5

- For every graph $G$ of order $n$,
  - $\chi(G) \geq \omega(G)$ and $\chi(G) \geq n/\beta(G)$. 
Theorem 10.7

• For every graph
  \[ \chi(G) \leq 1 + \Delta(G). \]
Theorem 10.8 (Brooks’ Theorem)

• For every connected graph $G$ that is not an odd cycle or a complete graph,
  – $X(G) \leq \Delta(G)$. 