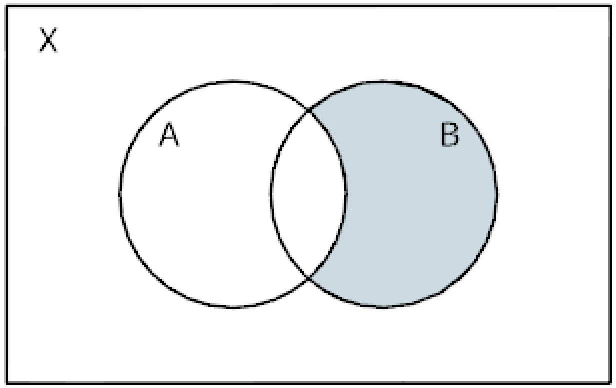
问题反馈

13/12/6

Paradox

- Barber shaves all those who do not shave themselves.
- Collection D includes all collections do not include themselves as elements.
- Be careful when creating a defining property.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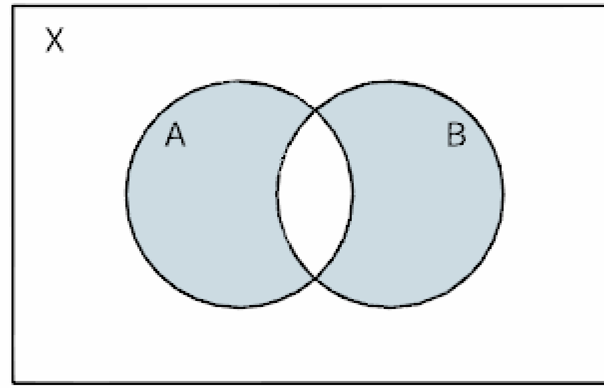
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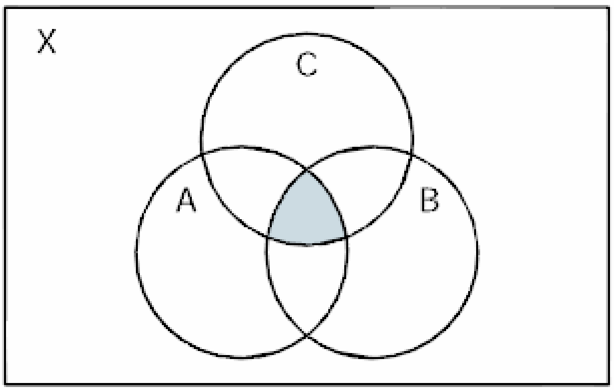
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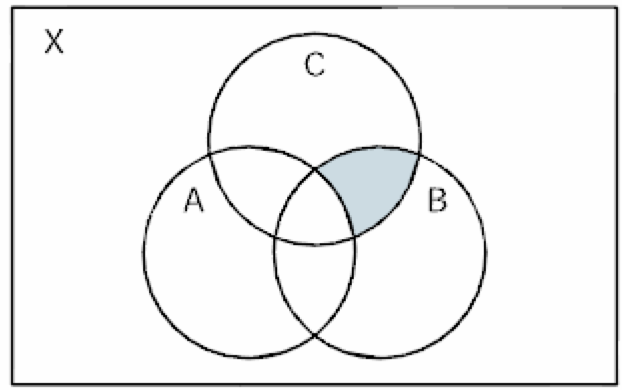
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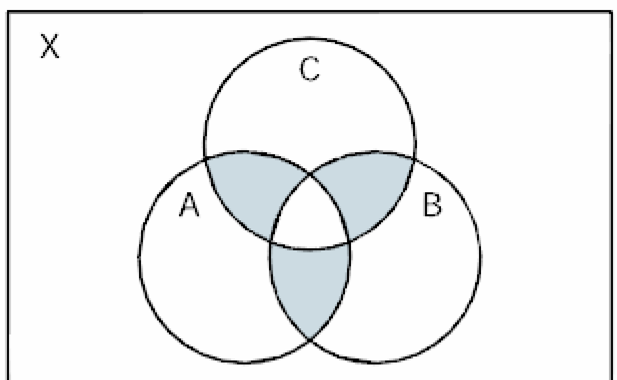
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Problem 7.8.

Consider the following sets:

(i) $(A \cap B) \setminus (A \cap B \cap C)$,

(ii) $A \cap B \setminus (A \cap B \cap C)$,

(iii) $A \cap B \cap C^c$,

(iv) $(A \cap B) \setminus C$, and

(v) $(A \setminus C) \cap (B \setminus C)$.

(a) Which of the sets above are written ambiguously, if any?

Problem 7.11.

Prove or give a counterexample for the following statement.

Let X be the universe and $A, B \subseteq X$. If $A \cap Y = B \cap Y$ for all $Y \subseteq X$, then $A = B$.

Problem 8.1.

Consider the intervals of real numbers given by $A_n = [0, 1/n)$, $B_n = [0, 1/n]$, and $C_n = (0, 1/n)$.

(a) Find $\bigcup_{n=1}^{\infty} A_n$, $\bigcup_{n=1}^{\infty} B_n$, and $\bigcup_{n=1}^{\infty} C_n$.

(b) Find $\bigcap_{n=1}^{\infty} A_n$, $\bigcap_{n=1}^{\infty} B_n$, and $\bigcap_{n=1}^{\infty} C_n$.

(c) Does $\bigcup_{n\in\mathbb{N}} A_n$ make sense? Why or why not?

The **natural numbers** $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.
The **integers** $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$.
The **positive integers** $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$.
The **real numbers** $\mathbb{R}$.
The **plane** $\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}\}$.
For $n \in \mathbb{Z}^+$, **Euclidean** n**-space** $\mathbb{R}^n = \{(x_1, x_2, \ldots, x_n) : x_j \in \mathbb{R} \text{ for } j = 1, 2, \ldots, n\}$.
The **rational numbers** $\mathbb{Q} = \{p/q : p, q \in \mathbb{Z} \text{ and } q \neq 0\}$.
The **complex numbers** $\mathbb{C} = \{a + bi : i^2 = -1 \text{ and } a, b \in \mathbb{R}\}$.

Problem 8.4.

Prove or give a counterexample: Let $\{A_n : n \in \mathbb{Z}^+\}$ and $\{B_n : n \in \mathbb{Z}^+\}$ be two indexed families of sets. If $A_n \subset B_n$ for all $n \in \mathbb{Z}^+$, then

$$\bigcap_{n=1}^{\infty} A_n \subset \bigcap_{n=1}^{\infty} B_n.$$

(Recall that $A \subset B$ means strict inclusion; that is, $A \subseteq B$ and $A \neq B$.)

Problem[♮] 8.11.

A collection of sets $\{A_\alpha : \alpha \in I\}$ is said to be a **pairwise disjoint collection** if the following is satisfied: For all $\alpha, \beta \in I$, if $A_\alpha \cap A_\beta \neq \emptyset$, then $A_\alpha = A_\beta$. Suppose that each set A_α is nonempty.

(a) Give an example of pairwise disjoint sets $A_1, A_2, A_3, \ldots$.

(b) What is the contrapositive of "if $A_\alpha \cap A_\beta \neq \emptyset$, then $A_\alpha = A_\beta$"?

(c) What is the converse of "if $A_\alpha \cap A_\beta \neq \emptyset$, then $A_\alpha = A_\beta$"?

(d) If $\{A_\alpha : \alpha \in I\}$ is a pairwise disjoint collection, does the assertion you found in (b) hold for all α and β in I?

(e) If the assertion that you found in (b) holds for all α and β in I, is $\{A_\alpha : \alpha \in I\}$ a pairwise disjoint collection?

(f) If $\{A_\alpha : \alpha \in I\}$ is a pairwise disjoint collection of sets, does it follow that $\bigcap_{\alpha \in I} A_\alpha = \emptyset$?

(g) If $\bigcap_{\alpha \in I} A_\alpha = \emptyset$, is $\{A_\alpha : \alpha \in I\}$ necessarily a pairwise disjoint collection of sets?

Problem 9.4.
Show that $A \subseteq B$ if and only if $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

Problem 9.13.
Suppose A, B, C, and D are four sets. If $A \times B \subseteq C \times D$, must $A \subseteq C$ and $B \subseteq D$? Why or why not?

Problem 9.16.
This problem introduces rigorous definitions of an ordered pair and Cartesian product. Let A be a set and $a, b \in A$. We define the ordered pair of a and b with first coordinate a and second coordinate b as

$$(a, b) = \{\{a\}, \{a, b\}\}.$$

Using this definition prove the following.

(a) If $(a, b) = (x, y)$, then $a = x$ and $b = y$.
(b) If $a \in A$ and $b \in B$, then $(a, b) \in \mathcal{P}(\mathcal{P}(A \cup B))$.

Now we are able to define the Cartesian product of the two sets A and B as the set

$$A \times B = \{x \in \mathcal{P}(\mathcal{P}(A \cup B)) : x = (a, b) \text{ for some } a \in A \text{ and some } b \in B\}.$$

(c) Use the above definitions to prove that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$.

This is a pretty complicated definition. It is also not our idea, but rather an idea that was born from axioms. P. Halmos' book, [31], is an excellent reference for this subject.

- (b) $a \in A$, $b \in B$
- $a, b \in A \cup B$;
- $\{a\}$, $\{a,b\}$ are subsets of $A \cup B$;
- $\{a\}, \{a,b\} \in P(A \cup B)$;
- $\{\{a\}, \{a,b\}\}$ is subset of $P(A \cup B)$;
- $\{\{a\}, \{a,b\}\} \in P(P(A \cup B))$

- (c) $(x,y) \in A \times B$, $x \in A$ and $y \in B$;
- since $A \leqq C$, $x \in C$, likewise, $y \in D$;
- $\{x\}, \{x,y\} \leqq C \cup D$
- $\{x\}, \{x,y\} \in P(C \cup D)$
- $\{\{x\}, \{x,y\}\} \leqq P(C \cup D)$
- $\{\{x\}, \{x,y\}\} \in P(P(C \cup D))$
- $(x,y) \in P(P(C \cup D))$
- $(x,y) \in C \times D$, by definition of $C \times D$.