A glimpse of Cantor's Paradox

Open Topic 1-8-2

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Naïve set theory

Theorem (Naïve set theory)

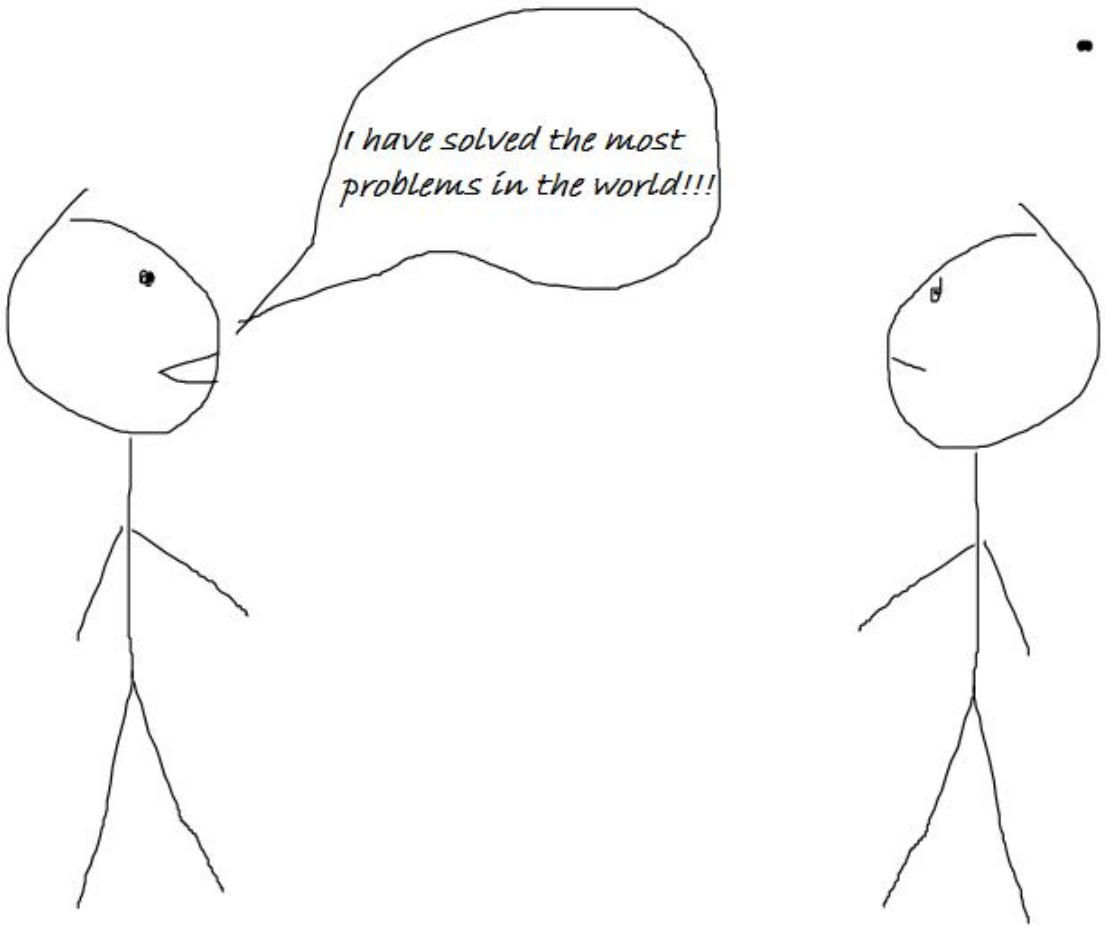
Set: The colleciton of elements that satisfise certain property.

NAIVE

Cantor's Paradox

Paradox

There is a greatest cardinal number.



Cardinal Number

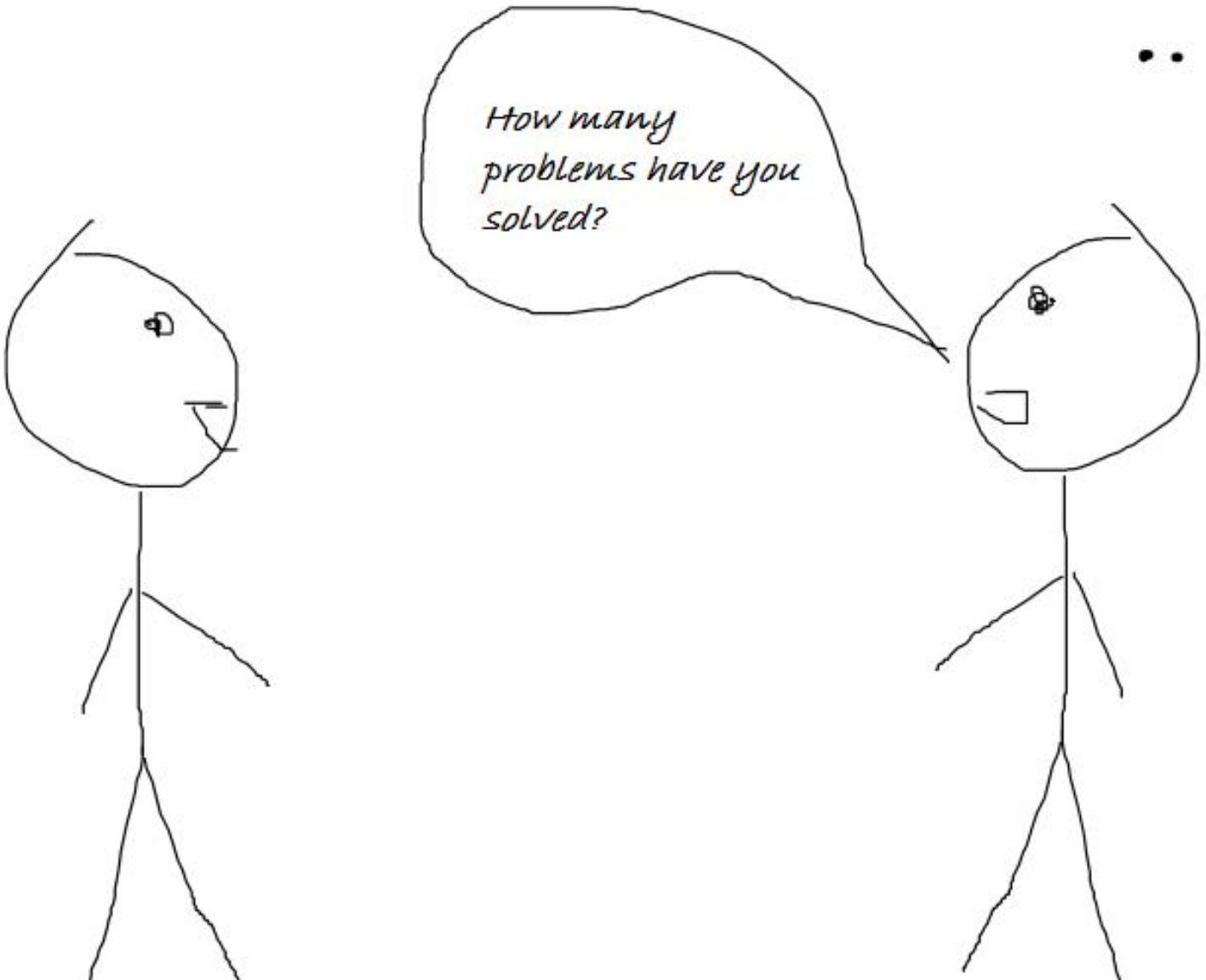
Definition (Cardinal Number)

Cardinal number is:
natural number used to measure the size of set.

Note

For a set S, denote that $|S|$ is the cardinality of S.
(See also: Von Neumann cardinal assignment)

Comic



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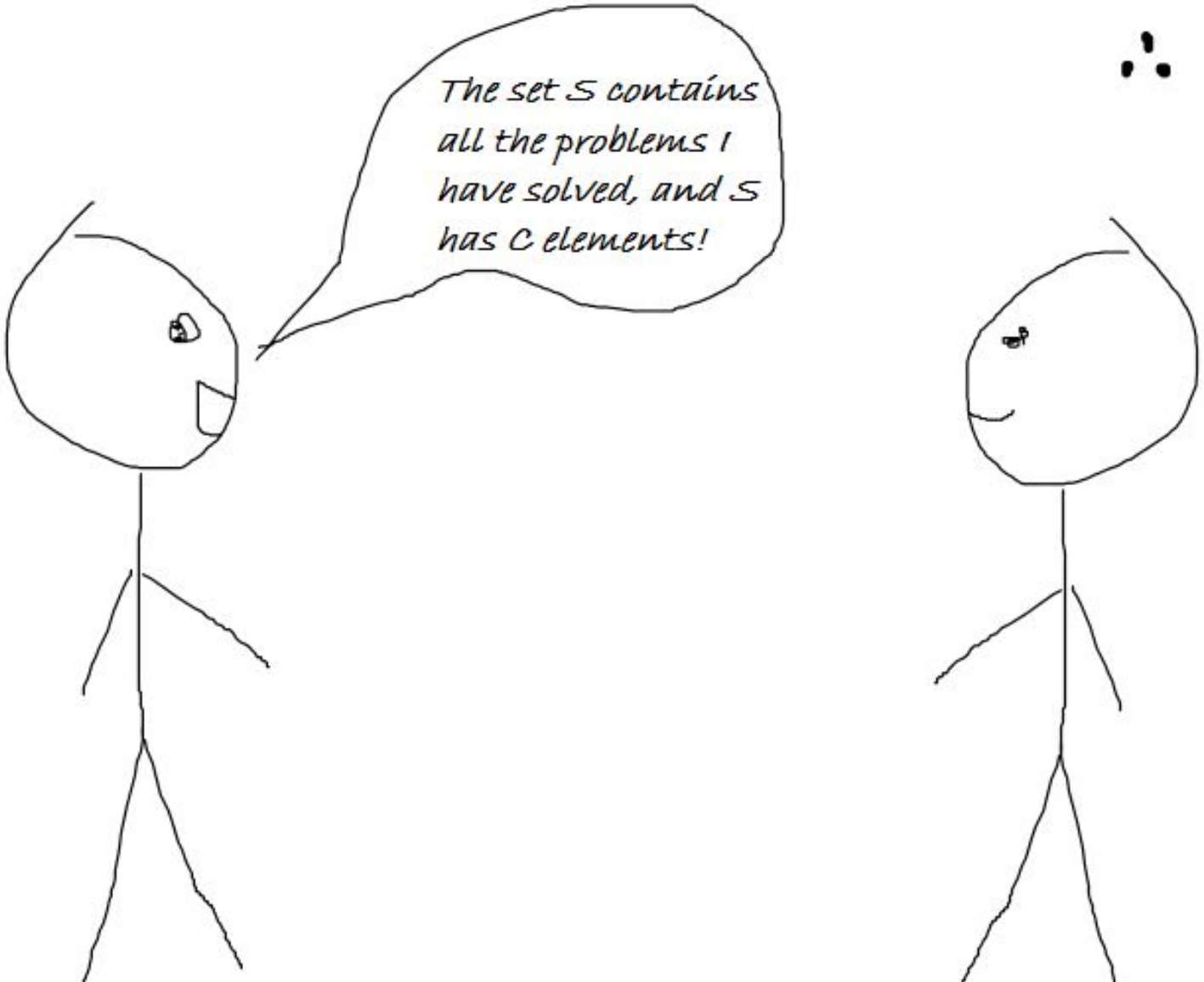

The set S contains all the problems I have solved, and S has C elements!

The Pigeonhole Principle

Theorem (The Pigeonhole Principle)

(i) $|A| > |B| \Rightarrow \forall f : A \to B,\ f$ *is not one-to-one.*

(ii) $|A| < |B| \Rightarrow \forall f : A \to B,\ f$ *is not onto.*

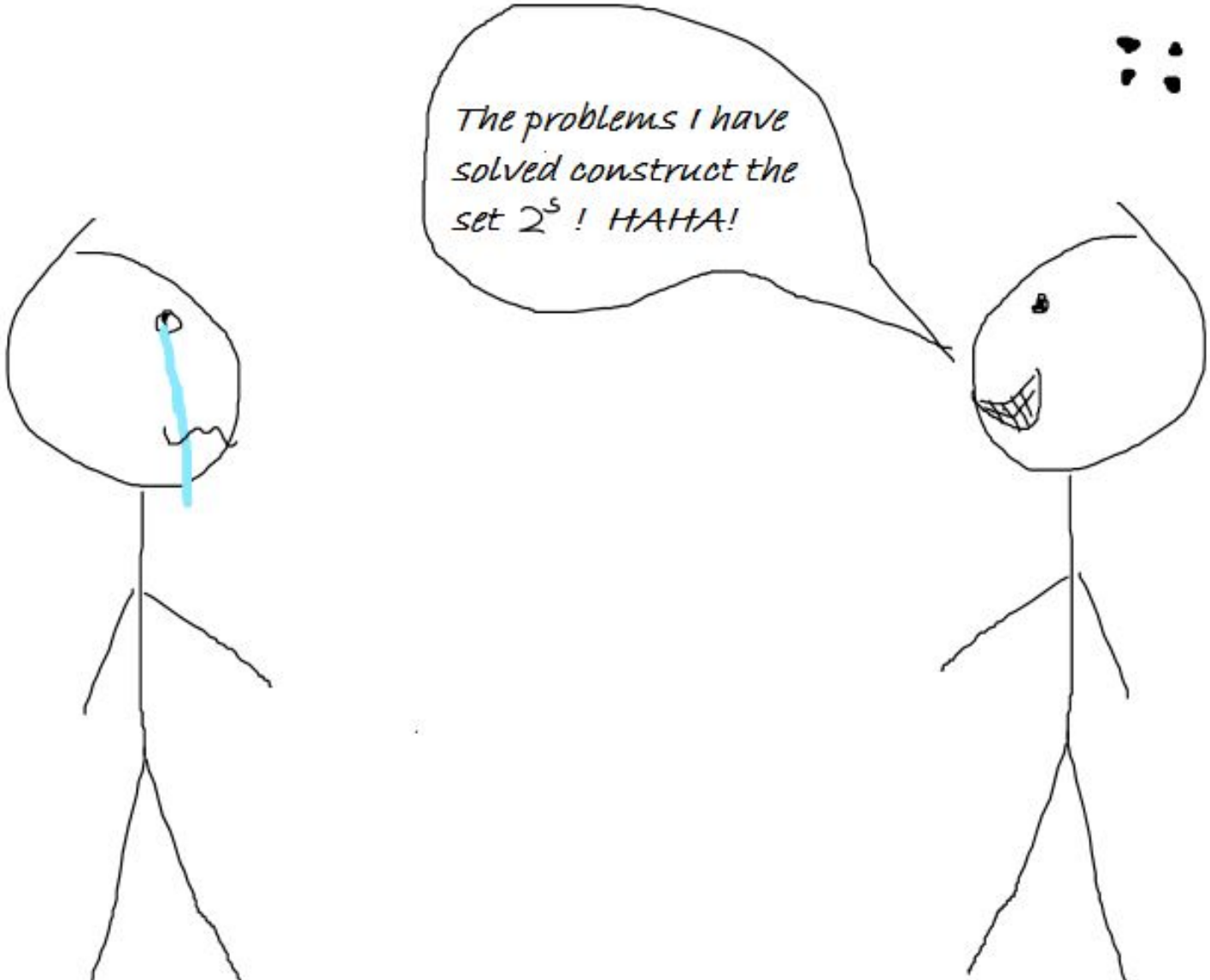
(iii) $\exists f : A \to B,\ f$ *is both one-to-one and onto* $\Rightarrow |A| = |B|$.

Cantor's Theorem

E.S. Theorem 24.4 (Cantor's Theorem)

Let S be a set, then $f : S \to 2^S(\mathcal{P}(S)) \Rightarrow f$ is not onto.

Comic



Disproof of Cantor's paradox

Disproof

(i) *There is a greatest cardinal number. Let it be* C *and we have the set* S *such that* $|S| = C$.

(ii) *Defined by the von Neumann formulation of cardinality, Since* S *is a set, then there exists its power set* 2^S.

(iii) *According to Cantor's Theorem,* $|S| < |2^S|$. $\Rightarrow\Leftarrow$ $\boxtimes$

Cantor's Paradox

Theorem

There is no greatest cardinal number.