A glimpse of Cantor’s Paradox
Open Topic 1-8-2

谢谢

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Naïve set theory

Theorem (Naïve set theory)

Set: The collection of elements that satisfy certain property.
Cantor’s Paradox

Paradox

There is a greatest cardinal number.

"I have solved the most problems in the world!!!"
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Cardinal Number

Definition (Cardinal Number)

*Cardinal number is:
natural number used to measure the size of set.*

Note

*For a set $S$, denote that $|S|$ is the cardinality of $S$.
(See also: Von Neumann cardinal assignment)*
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Comic

How many problems have you solved?
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Comic

The set $S$ contains all the problems I have solved, and $S$ has $\mathcal{C}$ elements!
The Pigeonhole Principle

Theorem (The Pigeonhole Principle)

(i) $|A| > |B| \implies \forall f : A \rightarrow B, \ f \ is \ not \ one-to-one.$
(ii) $|A| < |B| \implies \forall f : A \rightarrow B, \ f \ is \ not \ onto.$
(iii) $\exists f : A \rightarrow B, \ f \ is \ both \ one-to-one \ and \ onto \ \implies |A| = |B|.$
Cantor’s Theorem

E.S. Theorem 24.4 (Cantor’s Theorem)

Let $S$ be a set, then $f : S \rightarrow 2^S(\mathcal{P}(S)) \Rightarrow f$ is not onto.
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Comic

The problems I have solved construct the set $2^\omega$! HAHA!
Disproof of Cantor’s paradox

Disproof

(i) There is a greatest cardinal number. Let it be $C$ and we have the set $S$ such that $|S| = C$.

(ii) Defined by the von Neumann formulation of cardinality, Since $S$ is a set, then there exists its power set $2^S$.

(iii) According to Cantor’s Theorem, $|S| < |2^S|$. $\Rightarrow \Leftarrow$
Theorem

*There is no greatest cardinal number.*