计算机问题求解 – 论题2-15

- 图的基本概念

课程研讨

- GZ第1章、第2.1-2.2节、第3.1节

问题1：图论中的术语

- 图 (graph)、顶点 (vertex/node)、边 (edge)
- 相邻 (adjacent)、关联 (incident)、邻居 (neighbor)
- 阶 (order)、边数 (size)
  - 边数的上下界是多少？（与阶为单位）

- 简单图 (simple graph)
- 平行边/重边 (parallel/multiple edges)、环边 (loop)

- 有向图
  - 有向边 (arc/directed edge)、有向图 (digraph)
  - 定向图 (oriented graph)、底图 (underlying graph)*

问题1：图论中的术语 (续)

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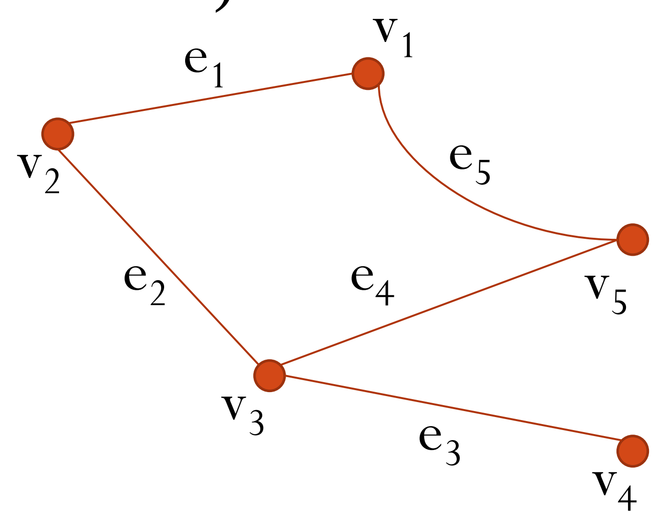
- 连通 (connected)、不连通 (disconnected)
- 连通分支 (component)
  - *n*个顶点的连通分支至少含几条边？
- 距离(distance)、直径 (diameter)



问题1：图论中的术语 (续)

- 度 (degree)
  - 为什么奇度顶点的个数总是偶数？
- 孤立点 (isolated vertex)
- 叶子 (leaf)
- 最小度 (δ)、最大度 (Δ)

- 有向图
  - 入度 (indegree)、出度 (outdegree)

问题1：图论中的术语 (续)

- 零图 (null graph)*
- 空图 (empty graph)
- 平凡图 (trivial graph)
- 完全图 (complete graph)
  - 完全图的边数和阶是什么关系？
- r-正则图 (r-regular graph)
  - 你能构造一个阶为5的3-正则图吗？
- 二部图/二分图 (bipartite graph)
  - 为什么不含奇圈是二部图的充要条件？
- 完全二部图 (complete bipartite graph)
  - 星 (star)
- k-部图 (k-partite graph)

问题1：图论中的术语 (续)

- 子图 (subgraph)、真子图 (proper subgraph)
- 生成子图 (spanning subgraph)
- 导出子图 (induced subgraph)

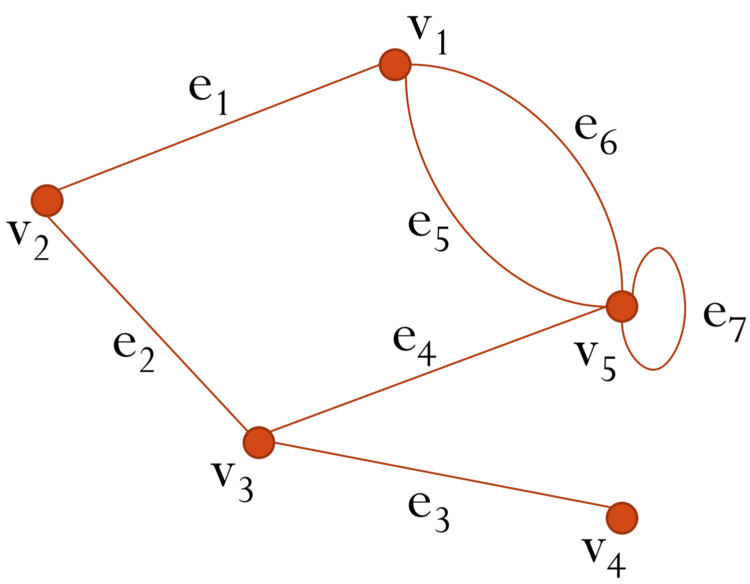
- 同构 (isomorphism)
  - 为什么同构是一种等价关系？

- 补图 (complement)
- 自补图 (self-complementary)
  - 你能举出例子吗？
- 并 (union)
- 联/连接 (join)
- 笛卡儿积 (Cartesian product)

问题1：图论中的术语 (续)

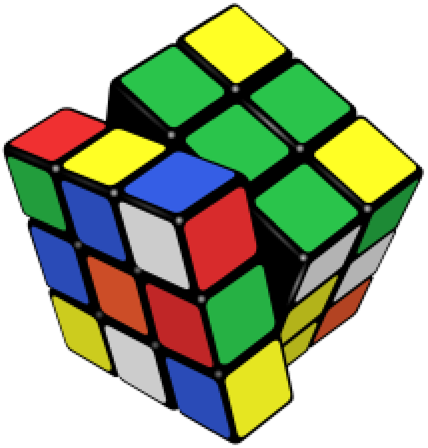
- 图的邻接矩阵表示

	v_1	v_2	v_3	v_4	v_5
v_1		1			2
v_2	1		1		
v_3		1		1	1
v_4			1		
v_5	2		1		1



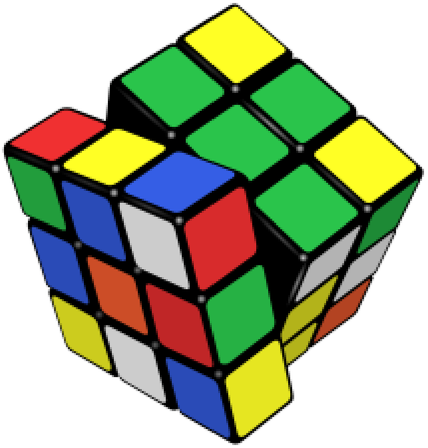
问题2：用图来建模问题

- 解决给定的魔方，最快需要多少步？
- 你能构造一个需要解决步数最多的魔方吗？



问题2：用图来建模问题 (续)

- 魔方状态转换图
  - 阶是多少？
  - 是连通图吗？
  - 是正则图吗？



问题2：用图来建模问题 (续)

- The suspects
  - Severe acute respiratory syndrome (SARS), an atypical pneumonia of unknown aetiology, was recognized as a global threat in mid-March 2003. To minimize transmission to others, the best strategy is to separate the suspects from others.
  - In the Not-Spreading-Your-Sickness University (NSYSU), there are many student groups. Students in the same group intercommunicate with each other frequently, and a student may join several groups. To prevent the possible transmissions of SARS, the NSYSU collects the member lists of all student groups, and makes the following rule in their standard operation procedure (SOP).
    - Once a member in a group is a suspect, all members in the group are suspects.
  - However, they find that it is not easy to identify all the suspects when a student is recognized as a suspect. Your job is to write a program which finds all the suspects.

问题2：用图来建模问题 (续)

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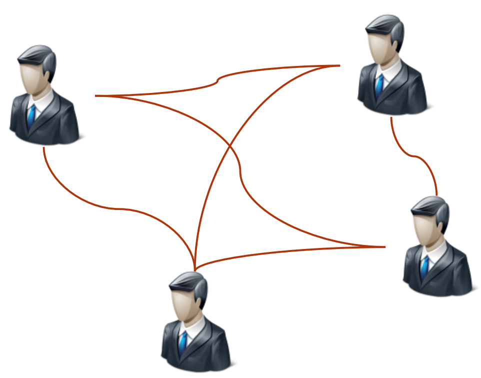
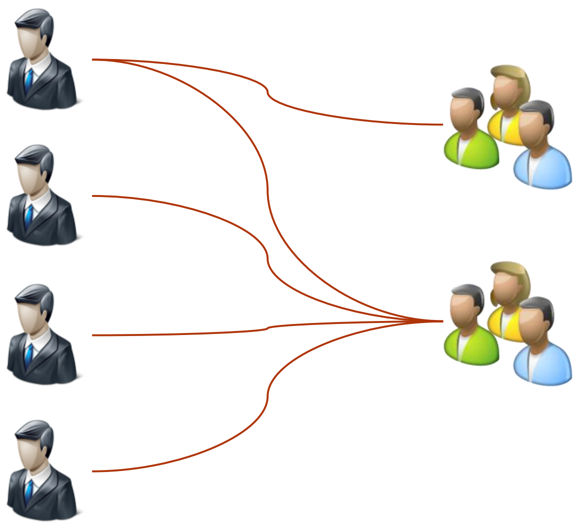
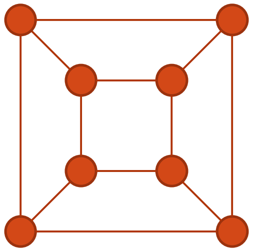
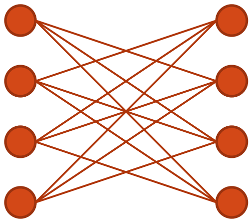




问题3：图论中的常用证明方法

- 构造法，例如：
  - 证明以下两个图同构。





问题3：图论中的常用证明方法 (续)

- 极端情况法（为自己新增一个条件），例如：
  - 证明：最小度为2的简单图必有圈，最小度为3的简单图必有偶圈。

问题3：图论中的常用证明方法 (续)

- 反证法，例如：
  - 任何简单图必有至少两个顶点具有相等的度。
  - 若G是简单图且 $\delta(G) \geq k$，则G有长为至少k的路。

问题3：图论中的常用证明方法 (续)

- 分情况讨论法，例如：
  - 如果图G不连通，则其补图必连通。

问题3：图论中的常用证明方法 (续)

- 数学归纳法，例如：
  - 设A是r阶图G的邻接矩阵，则A^n的第i行第j列元素$a_{ij}^{(n)}$等于G中从v_i到v_j的长度为n的途径的数目（$1 \le n < r$）。

问题3：图论中的常用证明方法 (续)

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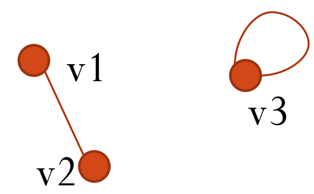
$$a_{ij}^{(n)} = \sum_{r=1}^{\nu} a_{ir}^{(n-1)} a_{rj}$$

问题4：图的集合表示

- 如何用集合的语言来表示一个无向图？你能想到几种方式？
  - 方法1
    - V={v1, v2, …}
    - E={e1, e2, …}
    - endpoints(e1)={v1, v2}, endpoints(e2)={v3, v3}, …
  - 方法2
    - V={v1, v2, …}
    - E={{v1, v2}, {v3}, …}
- 它们各有什么优缺点？



- 如何用集合的语言来表示一个有向图？
  - 方法1
    - V={v1, v2, …}
    - E={e1, e2, …}
    - tail(e1)=v1, tail(e2)=v3, …
    - head(e1)=v2, head(e2)=v3, …
  - 方法2
    - V={v1, v2, …}
    - E={{{v1}, {v1, v2}}, {{v3}}, …}