带边标记的DFS算法

算法实现和正确性

计算机科学与技术系 李松原

Nov. 4th, 2020

边标记

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Tree Edges

Back Edges

Forward Edges

Cross Edges

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First Access!

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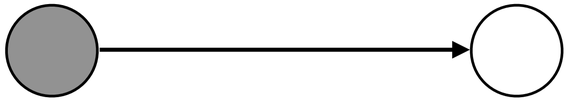
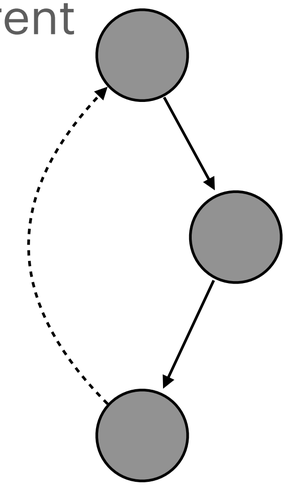


First Access!

A Gray Parent



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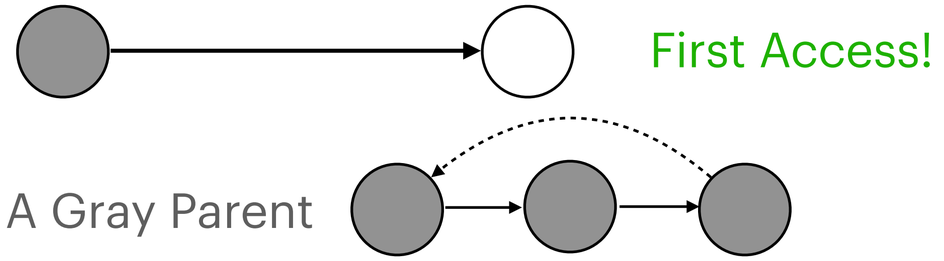

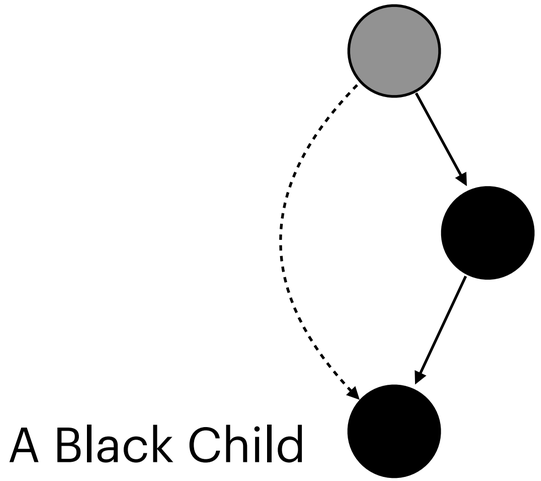

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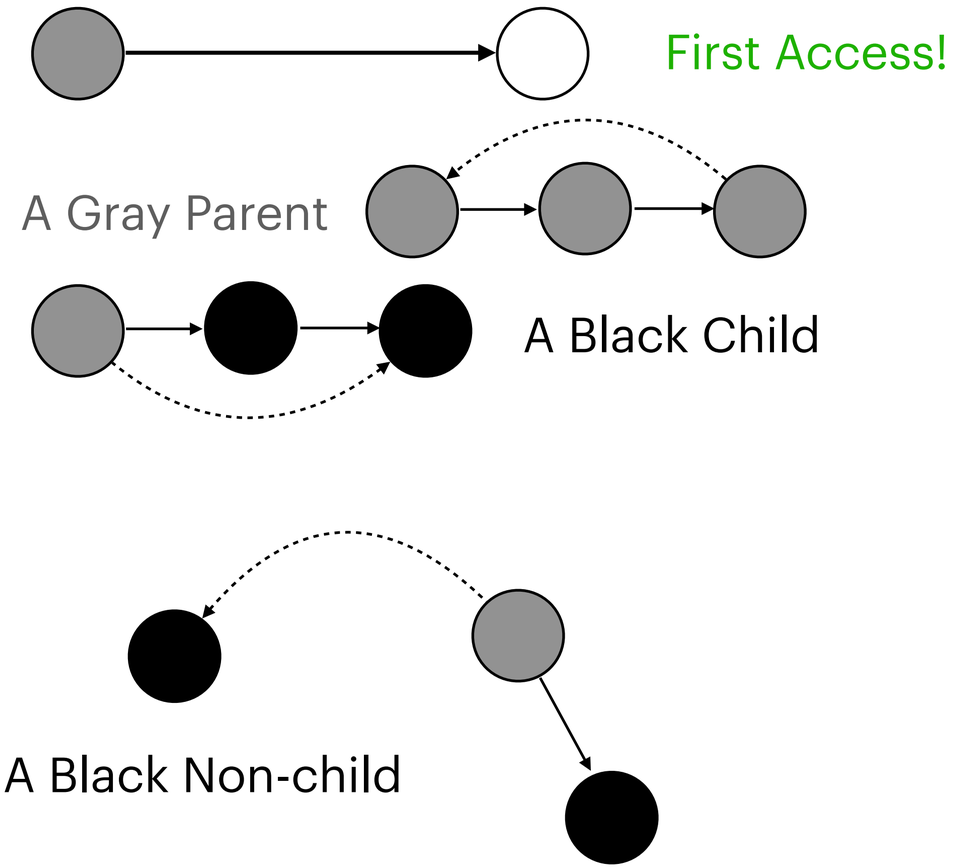

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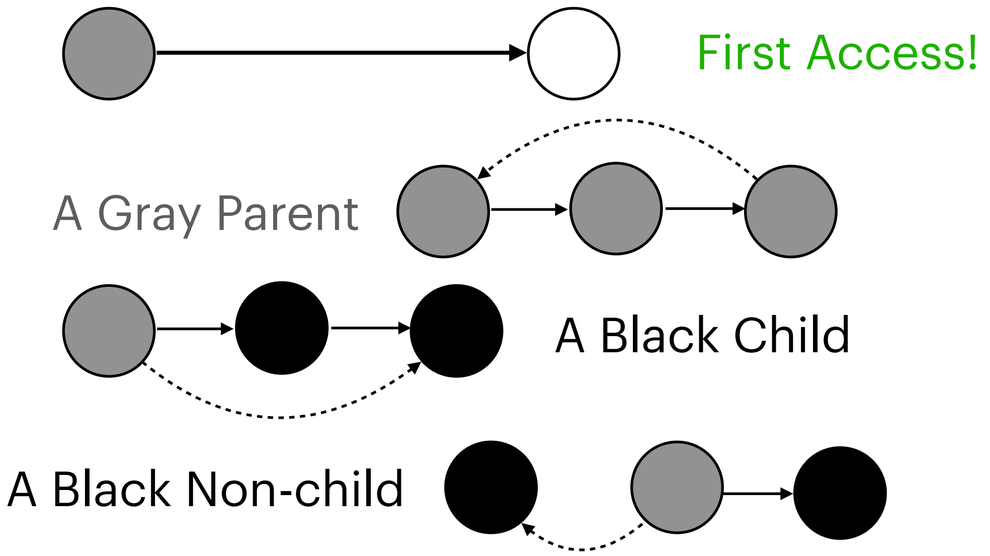

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procedure DFS-Visit(G, u)
    u.color = GRAY
    for each vertex v adjacent to u
        if v.color == WHITE
            DFS-Visit(G, v)
    u.color = BLACK
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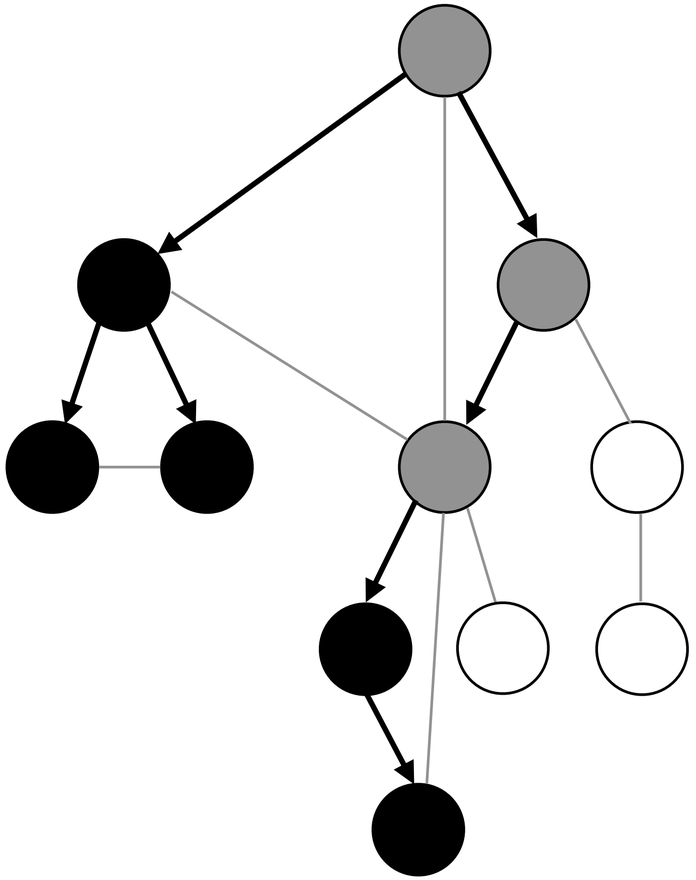

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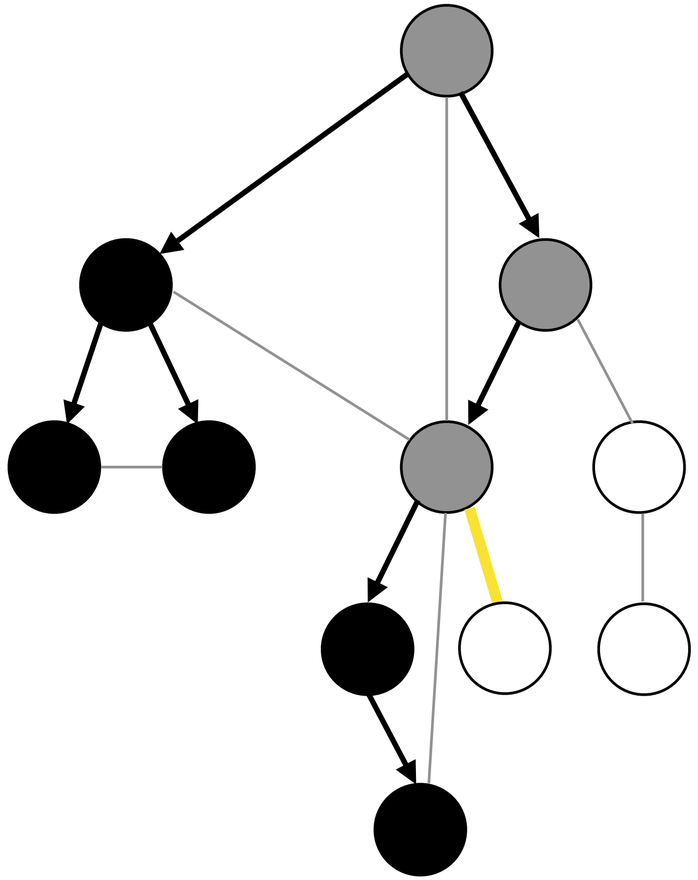

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procedure DFS-Visit(G, u)
    u.color = GRAY
    for each vertex v adjacent to u
        if v.color == WHITE
            Mark (u, v) as tree edge
            DFS-Visit(G, v)
    u.color = BLACK
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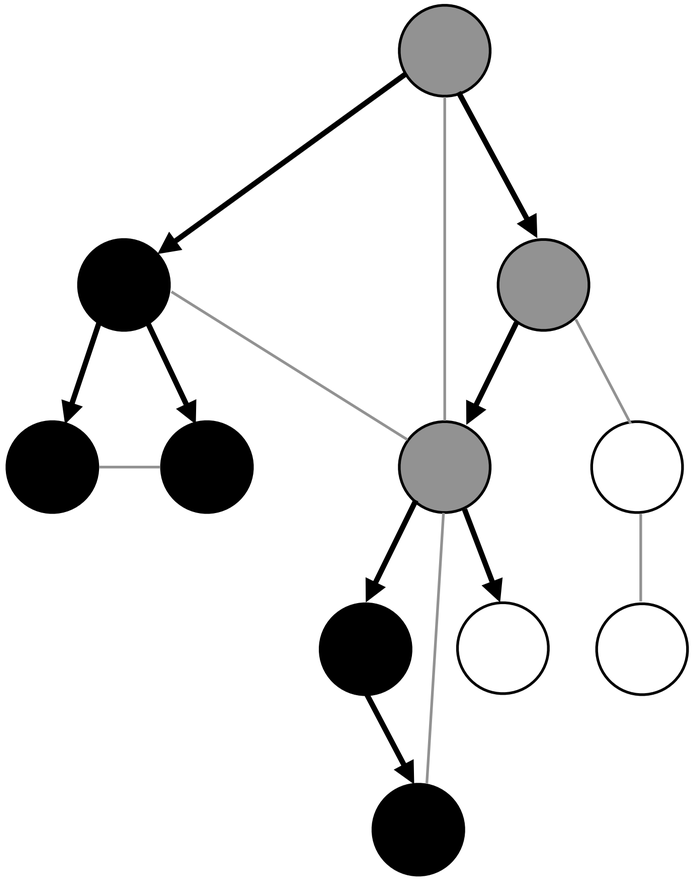

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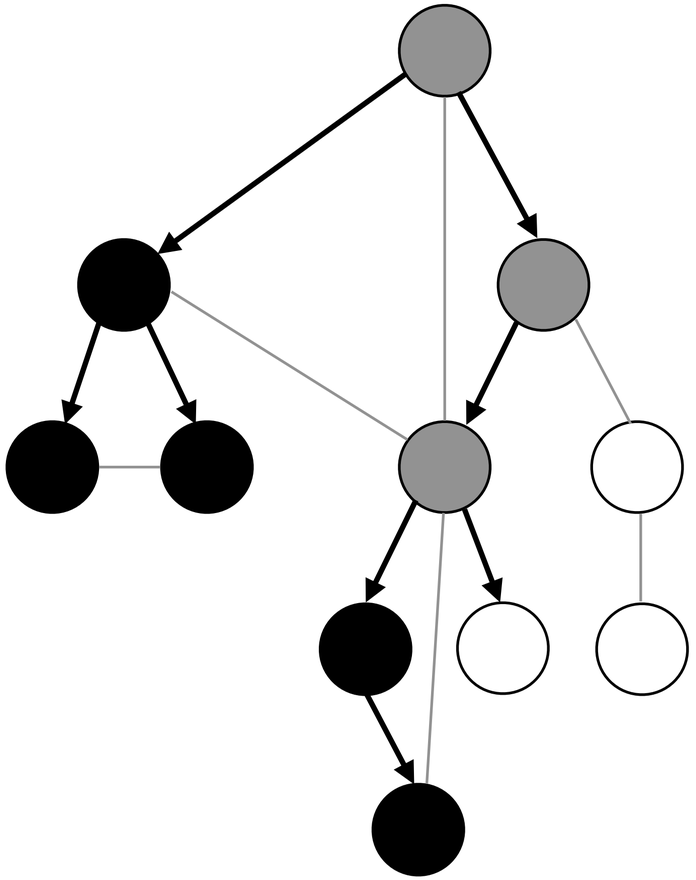

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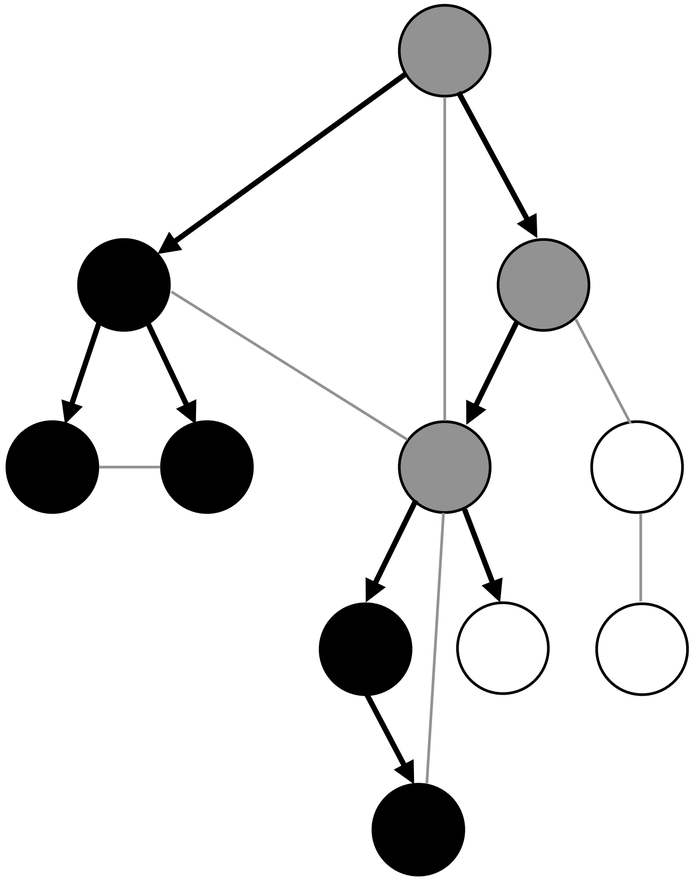

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   Mark (u, v) as forward/cross edge?
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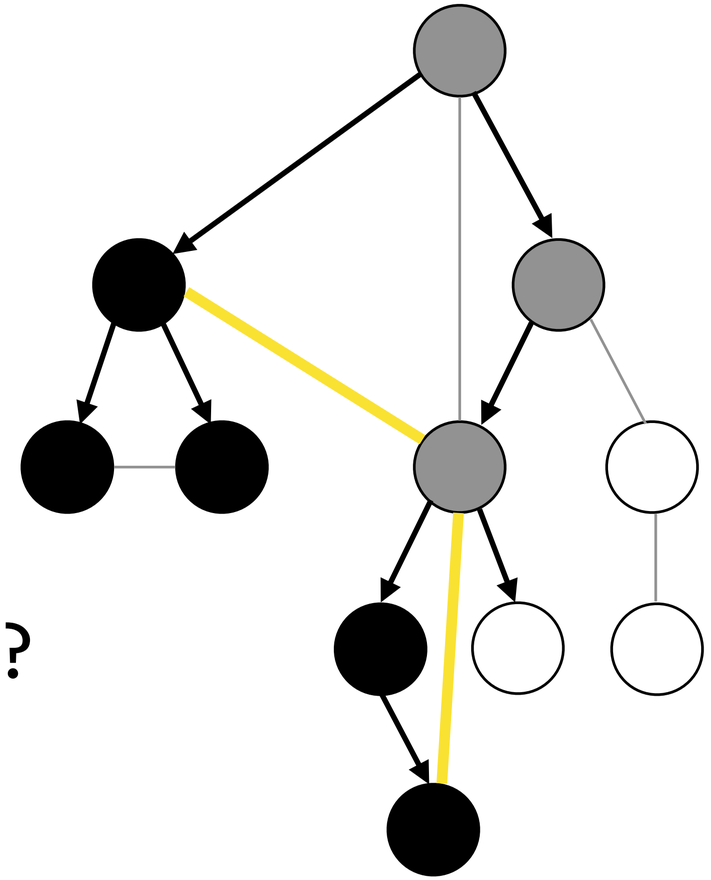

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else if v.color == BLACK
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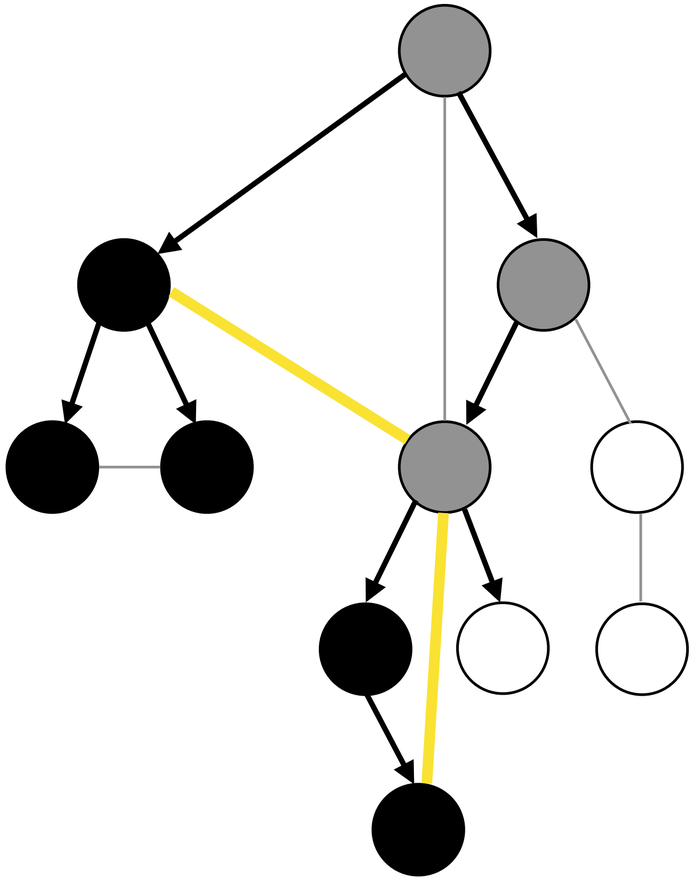

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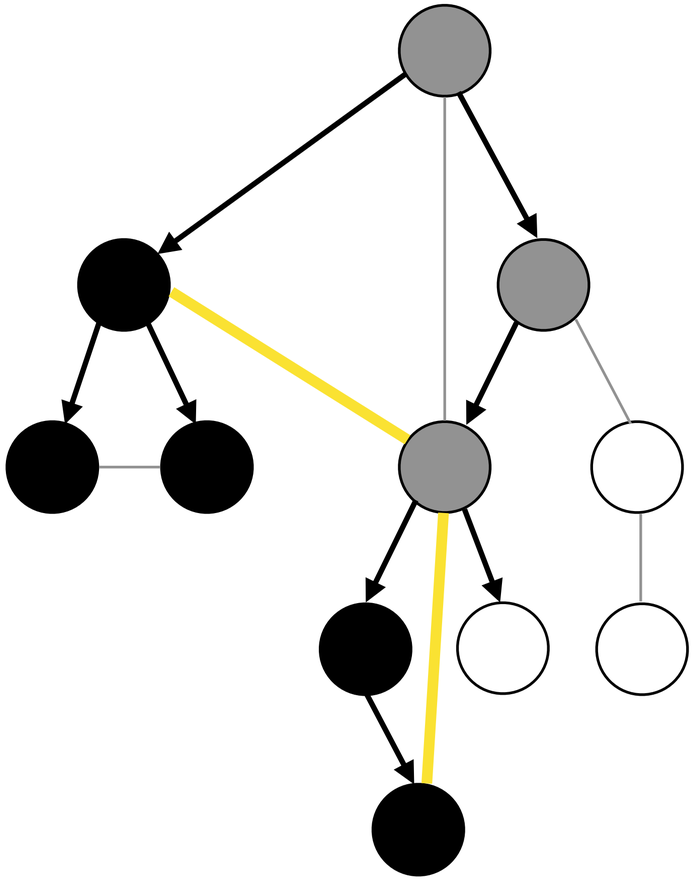
else if v.color == BLACK
   if v.f <= u.d
      Mark (u, v) as forward edge
   else
      Mark (u, v) as cross edge
```



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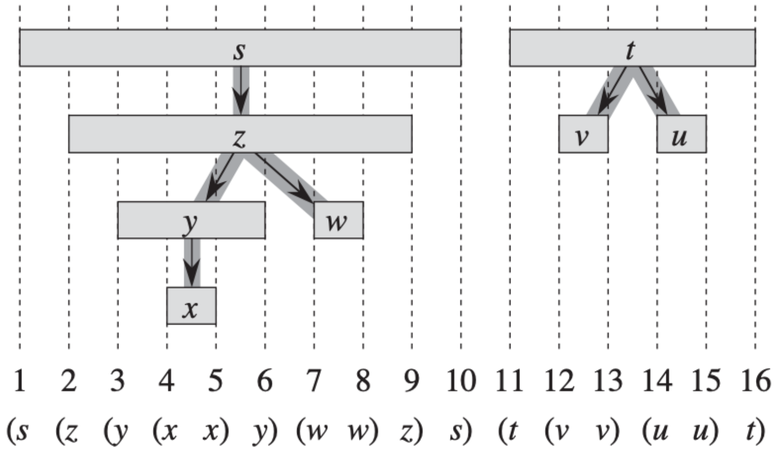

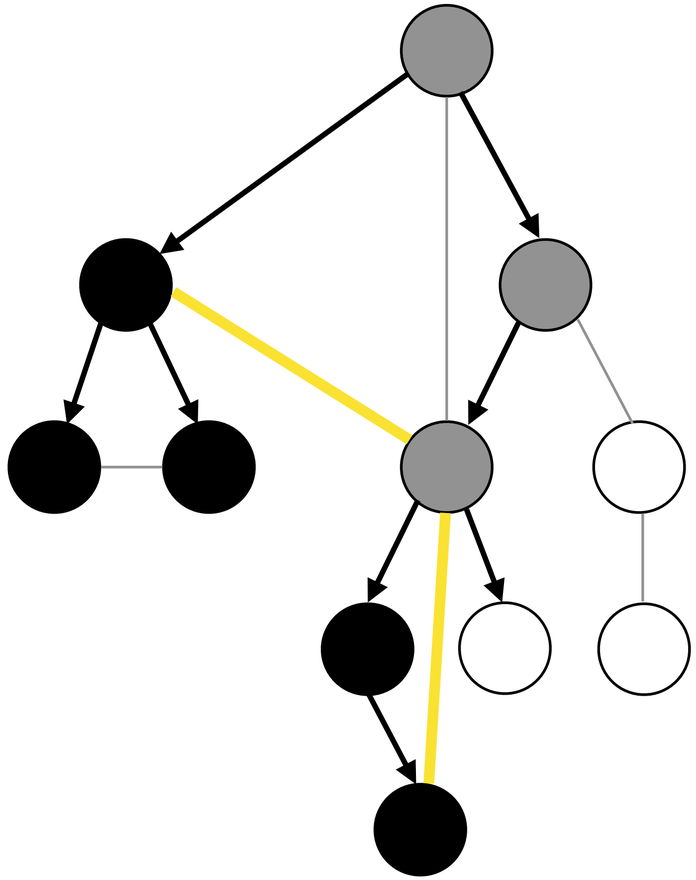

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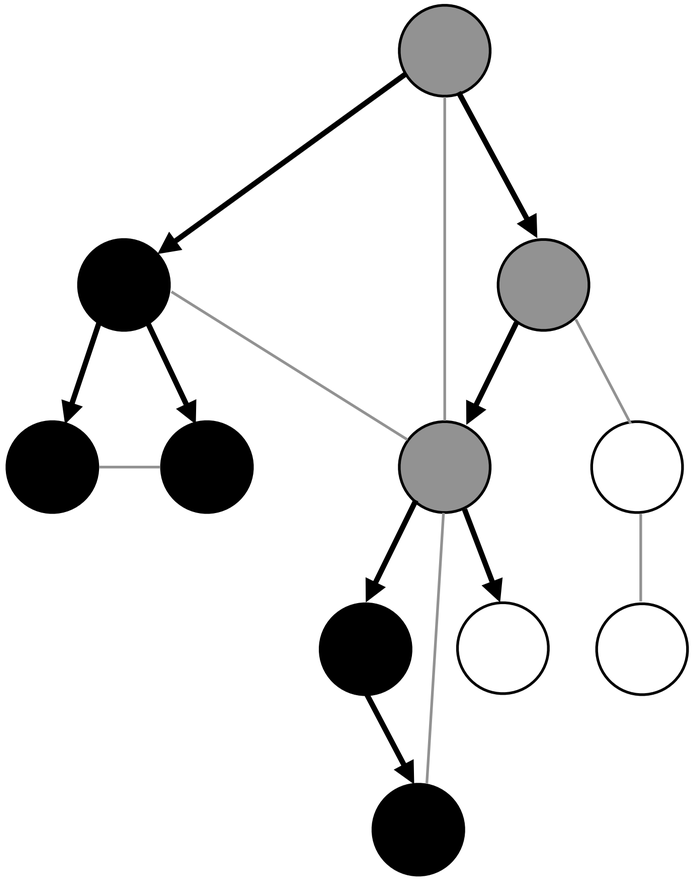

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正确性证明

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是否所有边都被标记?

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树边（Tree Edge）

标记的树边是否都为树边?

所有树边是否都被标记?

1. ***Tree edges*** are edges in the depth-first forest G_π. Edge (u, v) is a tree edge if v was first discovered by exploring edge (u, v).

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   Mark (u, v) as tree edge
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后向边（Back Edge）？

2. ***Back edges*** are those edges (u, v) connecting a vertex u to an ancestor v in a depth-first tree. We consider self-loops, which may occur in directed graphs, to be back edges.

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  Mark (u, v) as tree edge
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前向边（Forward Edge）？

3. ***Forward edges*** are those nontree edges (u, v) connecting a vertex u to a descendant v in a depth-first tree.

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4. ***Cross edges*** are all other edges. They can go between vertices in the same depth-first tree, as long as one vertex is not an ancestor of the other, or they can go between vertices in different depth-first trees.

无向图

无向图

Theorem 22.10
In a depth-first search of an undirected graph G, every edge of G is either a tree edge or a back edge.

Proof Let (u, v) be an arbitrary edge of G, and suppose without loss of generality that $u.d < v.d$. Then the search must discover and finish v before it finishes u (while u is gray), since v is on u's adjacency list. If the first time that the search explores edge (u, v), it is in the direction from u to v, then v is undiscovered (white) until that time, for otherwise the search would have explored this edge already in the direction from v to u. Thus, (u, v) becomes a tree edge. If the search explores (u, v) first in the direction from v to u, then (u, v) is a back edge, since u is still gray at the time the edge is first explored. ■

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if v.color == WHITE
  Mark (u, v) as tree edge
  DFS-Visit(G, v)
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  Mark (u, v) as back edge
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感谢聆听

Thanks for your listening!

Enjoy the rest of the week!