问题与反馈

2014-10-22

- 练习4.4 4.8 4.14 4.22 4.26 4.28 4.30 4.36

4.8

- Prove that if every vertex of a graph G has degree at least 2, then G contains a cycle.

(1) No cycle and connected→ a tree, contradicts total degree (2n-2 vs >= 2n);
(2) No cycle and disconnected → a forest, contradicts total degree (2n-2k vs >= 2n);

Proof: We shall prove the contrapositive, i.e., if G contains no cycles, then G has a vertex with degree less than 2. To this end, suppose that G has no cycles. Then G must be a forest. Let T be a component of G. If T is trivial, then T, and thus G, has a vertex of degree 0. If T is nontrivial, then T is a nontrivial tree and Theorem 4.3 implies that T has at least two end-vertices. These are of degree 1. Consequently, if G has no cycles, then G has at least one vertex with degree less than 2.//

4.26

- Prove that an edge e of a connected graph is a bridge if and only if e belongs to every spanning tree of G.

Proof:

⇒: [Contrapositive] Suppose that e does not belong to every spanning tree of G. Let T be a spanning tree that does not contain e. Then the tree T is a spanning subgraph of G - e. It follows from Theorem 4.2 that if u and v are any two vertices of G - e, then there is a unique u - v path in T. This is also a u - v path in G - e. Thus, G - e is connected and e is not a bridge.

⇐: [Contrapositive] Suppose e is not a bridge. Then G - e is connected, and Theorem 4.10 implies that G - e has a spanning tree T. Since V(G) = V(G - e), T is a spanning tree of G, a spanning tree that does not contain e.

4.30

- Let G be a connected weighted graph and T a minimum spanning tree of G. Show that T is a unique minimum spanning tree of G if and only if the weight of each edge e of G that is not in T exceeds the weight of every other edge on the cycle in T+e.

- → Suppose T is the unique MST, but there exists e’ not in T but has smaller or equal weight than every edge on the cycle in T+e’. Remove another different edge f on the cycle and we get T+e’-f, another ST with smaller or equal weights. Contradicts.