- 书面讲解
  - TC第4.1节练习5
  - TC第4.3节练习3、7
  - TC第4.4节练习2、8
  - TC第4.5节练习4
  - TC第4章问题1、3、4

# TC第4.1节练习5

```
int sequence(std::vector<int>& numbers)
{
        // Initialize variables here
        int max_so_far  = numbers[0], max_ending_here = numbers[0];
        size_t begin = 0;
        size_t begin_temp = 0;
        size_t end = 0;
        // Find sequence by looping through
        for(size_t i = 1; i < numbers.size(); i++)
        {
                // calculate max_ending_here
                if(max_ending_here < 0)
                {
                        max_ending_here = numbers[i];
                        begin_temp = i;
                }
                else
                {
                        max_ending_here += numbers[i];
                }
                // calculate max_so_far
                if(max_ending_here > max_so_far )
                {
                        max_so_far  = max_ending_here;
                        begin = begin_temp;
                        end = i;
                }
        }
        return max_so_far ;
}
```

动态规划

- max_so_far: A[1..j]上的最大值
- max_ending_here: 最大的A[i...j+1]
- 两者中较大者为A[1..j+1]上的最大值

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13 | –3 | –25 | 20 | –3 | –16 | –23 | 18 | 20 | –7 | 12 | –5 | –22 | 15 | –4 | 7 |

抛弃和为负数的前缀

# TC第4.3节练习3

- 大胆缩放、正确缩放

$$T(n) \geq 2\left(c\left\lfloor \frac{n}{2} \right\rfloor \lg \left\lfloor \frac{n}{2} \right\rfloor\right) + n \geq 2c\frac{n-1}{2}\lg\frac{n}{4} + n$$

$$= cn\lg n - c\lg n - 2cn + 2c + n = cn\lg n - \frac{1}{4}\lg n + \frac{1}{2}n + \frac{1}{2} \geq cn\lg n$$

(c=1/4)

- 不要忘记boundary condition

# TC第4.3节练习7

如果欲证$T(n) \le cn^{\log_3 4}$

$T(n) \le \ldots \le cn^{\log_3 4} + n \ge cn^{\log_3 4}, \quad \text{fails}$

改为欲证$T(n) \le cn^{\log_3 4} - \frac{1}{4}n$

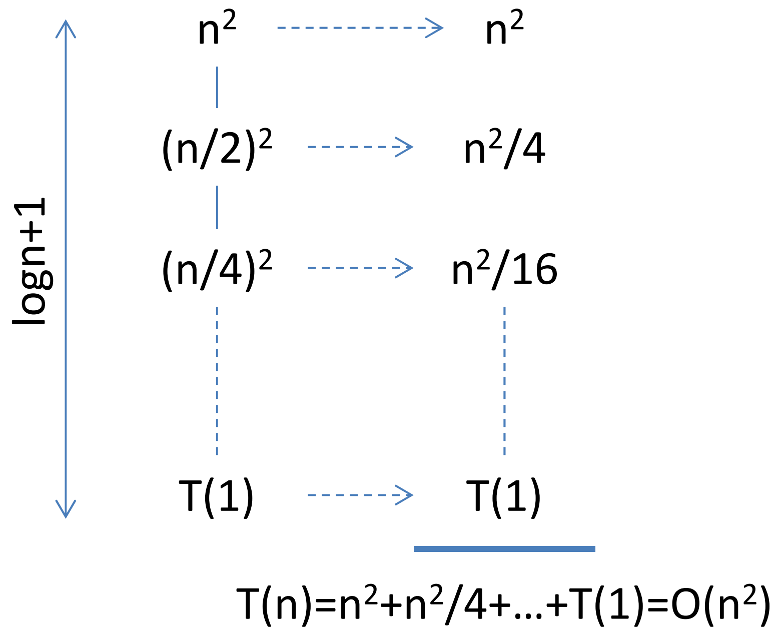
$T(n) \le \ldots \le cn^{\log_3 4} + n - n = cn^{\log_3 4}$，　这样对吗？

改为欲证$T(n) \le cn^{\log_3 4} + dn$

$T(n) \le \ldots \le cn^{\log_3 4} + \left(\frac{4}{3}d + 1\right)n \le cn^{\log_3 4} + dn$，　只要$\frac{4}{3}d + 1 \le d$

# TC第4.4节练习2

- recursion tree的组成元素缺一不可




- 如何证明$T(n)\in O(f(n))$？
  - 对于$n>n_0$，$T(n)\le c_1f(n)$
- 如何证明$T(n)\in \Theta(f(n))$？
  - 对于$n>n_0$，$c_1f(n)\le T(n)\le c_2f(n)$
- 什么是substitution method？
  - 数学归纳法

# TC第4.5节练习4

- 因为$n^2 \le n^2 \lg n \le n^3$，所以$n^2 \lg n = n^{2+\varepsilon}$。这个逻辑对吗？

- $\lg n \in o(n^{任意正数})$

# TC第4章问题3b

- 因为$n/\lg n<n$，所以$n/\lg n=n^{1-\varepsilon}$。这个逻辑对吗？
- $n/\lg n=n^{1-\varepsilon}$ → $1/\lg n=n^{-\varepsilon}$ → $\lg n=n^{\varepsilon}$ → $\lg n\in\Theta(n^{\varepsilon})$ → 与$\lg n\in o(n^{\text{任意正数}})$矛盾

# TC第4章问题3j

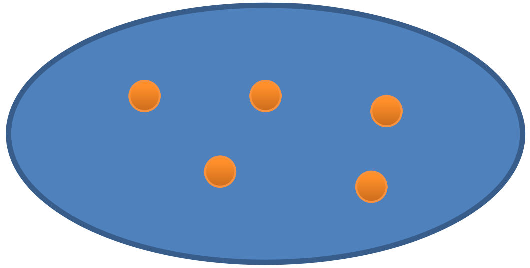
- $\Theta(n\lg\lg n)$
- 数学归纳法很容易证明

- 但是，怎么猜出这个结果？

- 教材讨论
  - CS第5章第1、2、3、4节

# 问题1： probability

- 你理解这些概念了吗？
  - Sample space
  - Element
  - Event
  - Probability weight
  - Probability



- 你能基于这些概念解释probability distribution function的三个条件吗？

1. $P(A) \geq 0$ for any $A \subseteq S$.
2. $P(S) = 1$.
3. $P(A \cup B) = P(A) + P(B)$ for any two disjoint events $A$ and $B$.

# 问题1：probability (续)

- 在这些例子中，sample space、element、event分别是什么？
  - The probability of getting at least 1 head in 5 flips of a coin.
  - The probability of getting a total of 6 or 7 on the 2 dice.
  - The probability that all 3 keys hash to different locations (among 20).

- 你能给出它们的答案吗？

# 问题1：probability (续)

- 你理解uniform probability distribution了吗？

**Theorem 5.2** *Suppose $P$ is the uniform probability measure defined on a sample space $S$. Then for any event $E$,*

$$P(E) = |E|/|S|,$$

*the size of $E$ divided by the size of $S$.*

- 现在你能给出之前几题的答案了吗？
  - The probability of getting at least 1 head in 5 flips of a coin.
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- uniform probability distribution为计算带来了怎样的便利？

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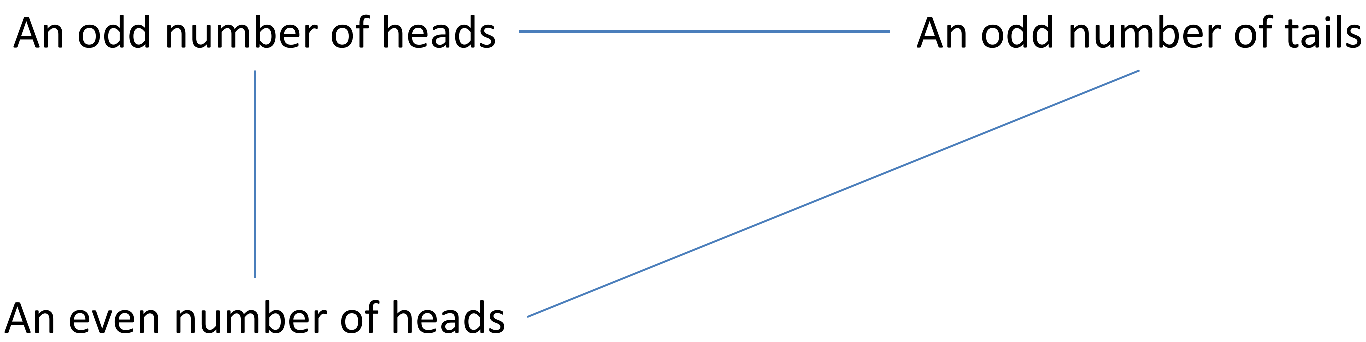
- uniform probability distribution为计算带来了怎样的便利？
  - probability → counting

# 问题1：probability (续)

- What is the probability of an odd number of heads in three tosses of a coin?（假设是uniform probability distribution）
  - 如何利用这个三角形快速求解？



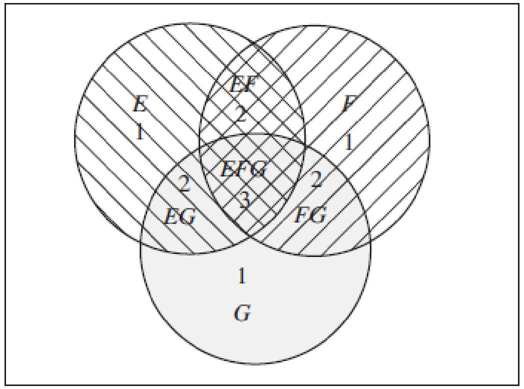

- 如果不是uniform probability distribution，怎么办？

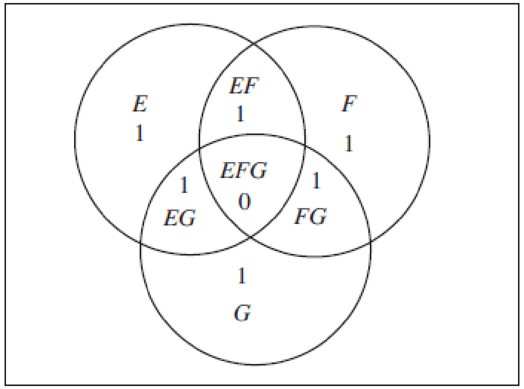
# 问题1： probability (续)

Are the following two events equally likely? Event 1 consists of drawing an ace and a king when you draw two cards from among the thirteen spades in a deck of cards and event 2 consists of drawing an ace and a king when you draw two cards from the whole deck.

# 问题2：the principle of inclusion and exclusion

- 你理解这两个图的含义了吗？
  - $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(F \cap G) + P(E \cap F \cap G)$




- 你读懂这个公式了吗？

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{k=1}^{n} (-1)^{k+1} \sum_{\substack{i_1, i_2, \ldots, i_k: \\ 1 \le i_1 < i_2 < \cdots < i_k \le n}} P\left(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k}\right)$$

# 问题2： the principle of inclusion and exclusion (续)

- How many functions from an *m*-element set *M* to an *n*-element set *N* map nothing to at least one element of *N*?
  - Sample space?
  - Element?
  - Event?

$$\left|\bigcup_{i=1}^{n} E_i\right| = \sum_{k=1}^{n}(-1)^{k+1} \sum_{\substack{i_1,i_2,\ldots,i_k:\\ 1\le i_1<i_2<\cdots<i_k\le n}} |E_{i_1}\cap E_{i_2}\cap\cdots\cap E_{i_k}|$$

$E_i$是什么？　　　在这里如何计算？

# 问题2：the principle of inclusion and exclusion (续)

- How many functions from an *m*-element set *M* to an *n*-element set *N* map nothing to at least one element of *N*?
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不映射到i的函数集合

$$\binom{m}{k}(m-k)^n$$

# 问题2： the principle of inclusion and exclusion (续)

- In how many ways may you distribute $k$ identical apples to $n$ children so that no child gets more than $m$?

# 问题2：the principle of inclusion and exclusion (续)

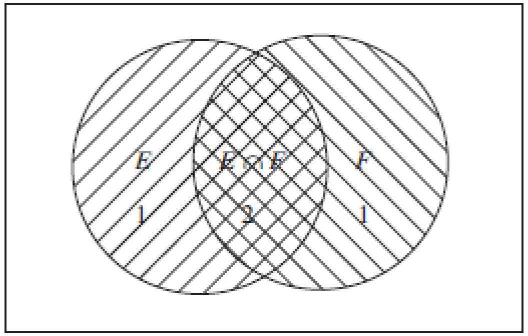
- In how many ways may you distribute *k* identical apples to *n* children so that no child gets more than *m*?

$$\binom{k+(n-1)}{n-1} - \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} \binom{k-(m+1)i+(n-1)}{n-1}$$

# 问题3：conditional probability

- 你能结合Venn图解释条件概率的定义吗？

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$



- 你能结合图解释独立性吗？

$$P(E|F) = P(E)$$

- 你能自己推导出这两个定理吗？

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}.$$

**Theorem 5.4** *Suppose $E$ and $F$ are events in a sample space. Then $E$ is independent of $F$ if and only if $P(E \cap F) = P(E)P(F)$.*

# 问题3：conditional probability (续)

$$P(x_i = a_i | x_1 = a_1, \ldots, x_{i-1} = a_{i-1}) = P(x_i = a_i)$$

- 你理解independent trials process了吗？

**Exercise 5.3-7** Suppose we draw a card from a standard deck of 52 cards, discard it (i.e. we do not replace it), draw another card and continue for a total of ten draws. Is this an independent trials process?

- 为什么这不是一个independent trials process？
- 为这个过程绘制tree diagram，并计算：第i张抽到梅花A的概率是多少？
- 如果是independent trails process，其tree diagram有什么特征？

# 问题3： conditional probability (续)

- A nickel, two dimes, and two quarters are in a cup. We draw three coins, one at a time, without replacement.
  - Draw the probability tree which represents the process.
  - Use the tree to determine the probability of getting a nickel on the last draw.
  - Use the tree to determine the probability that the first coin is a quarter, given that the last coin is a quarter.

# 问题4：random variables

- 你理解这些概念了吗？能自己举个例子吗？
  - Random variable
  - Expected value
  - $E(X+Y) = E(X) + E(Y)$

- 你能直观解释它们为什么相等吗？

$$E(X) = \sum_{i=1}^{k} x_i P(X = x_i)$$

$$E(X) = \sum_{s:s \in S} X(s) P(s)$$

# 问题4： random variables (续)

- How many sixes do we expect to see on top if we roll 24 dice?
- What is the expected number of times we need to roll two dice until we get a 7?

# 问题4：random variables (续)

- A student is taking a true-false test and guessing when he doesn't know the answer. We are going to compute a score by subtracting a percentage of the number of incorrect answers from the number of correct answers. That is, for some number y, the student's corrected score will be

  (number of corrected answers) – y(number of incorrect answers)

  When we convert this "corrected score" to a percentage score, we want its expected value to be the percentage of the material being tested that the student knows. How can we do this?