- 教材讨论
  - UD第6、7、8、9章

问题1：集合的描述

- 集合的两种描述方法分别有哪些优缺点？
  - extensional definition
    - {-1, 1}
    - {1}
  - intensional definition
    - $\{2n : n \in Z\}$
    - $\{(m,n) \in R^2 : y=0\}$

- 以下两个集合分别包含哪些元素？
  - $A = \{x : x \notin A\}$
  - $A = \{x : x \in A\}$

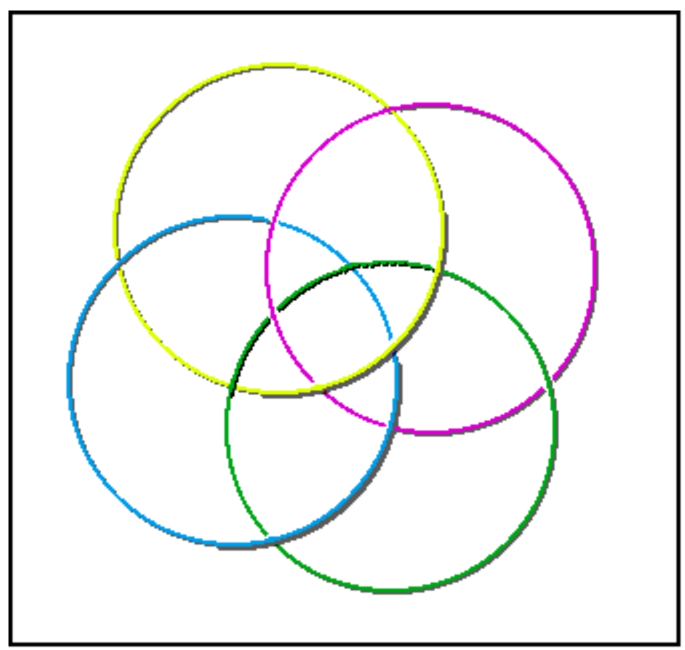
问题2：⊆ vs. ∈

- 假设a≠b
  - a ? {a, b}
  - {a} ? {a,b}
  - b ? {a,{a},{a,b}}
  - {a} ? {a,{a},{a,b}}
  - Ø ? {Ø}

- 赤兔马 ? 红马 ? 马
- 歼20 ? 战斗机 ? 飞机

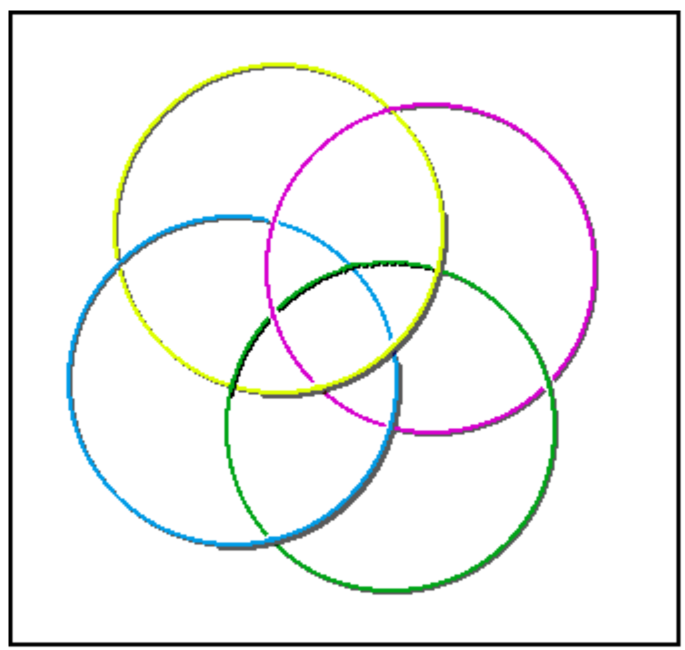
问题3：文氏图

- 文氏图能否直接用作与集合有关的证明？
- 你能利用以下文氏图证明$(A\cap C)\setminus B\subseteq D$吗？（你有没有发现什么问题？）



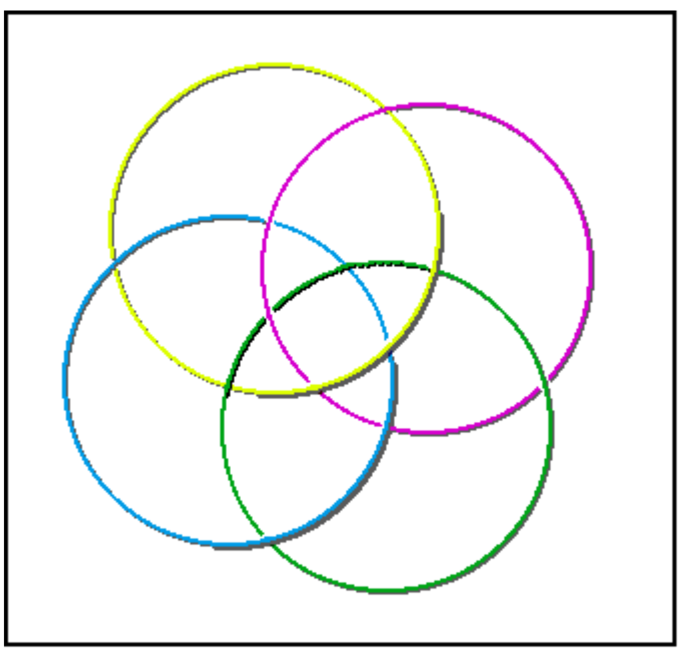
问题3：文氏图 (续)

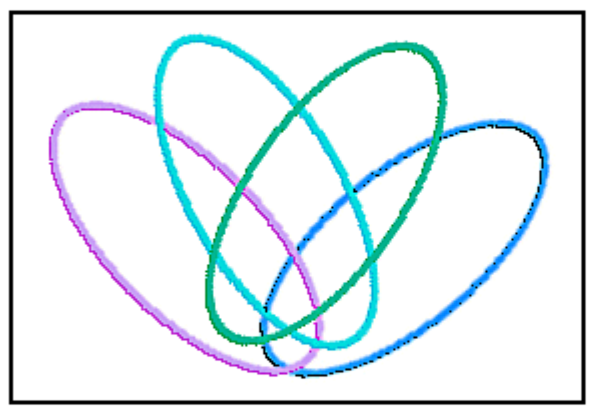
- 你能用四个正圆重新画出正确的文氏图吗？



问题3：文氏图 (续)

- 你能用四个正圆重新画出正确的文氏图吗？
  - 可以证明：画不出来
- 用四个椭圆可以





问题4： index set

- So suppose we have a set *I*, and suppose further that for each $\alpha \in I$ there is a set A_α corresponding to it.
  - 为什么要引入index set？
  - 你能从日常生活中举出使用index set的例子吗？
  - 你理解这些式子的含义了吗？

(a) $\bigcup_{x \in \mathbb{R}^+} (0, x)$;
(b) $\bigcup_{n \in \mathbb{N}} [0, n]$;
(c) $\bigcap_{n \in \mathbb{N}} [0, n]$.

问题5：有关集合的证明方法

- $A \cup \bigcup_{\alpha \in I} B_\alpha = A$ if and only if $\forall \alpha \in I, B_\alpha \subseteq A$.
  - 你能自上而下地（top-down）给出这个命题的详细证明过程吗？
    - 集合相等
    - 集合包含
    - if and only if

问题6：幂集

- 什么是幂集？
- P(A)中包含多少个元素？

- 以下两个命题是永真式吗？为什么？
  - $P(A \cap B) = P(A) \cap P(B)$
  - $P(A \cup B) = P(A) \cup P(B)$

问题7：笛卡尔积

- 什么是有序对？它和集合有什么区别？
- 你能用集合表示法来替代有序对并达到同样的效果吗？

- 什么是笛卡尔积？
- |A×B|中包含多少个元素？
- 什么是关系 (relation)？
- 你能给出日常生活中笛卡儿积和关系的例子吗？

问题8：Tips on writing mathematics

- 你理解P106中每条tip的意思吗？

- In mathematics, it is always important that the reader know what the variables stand for. This was true in algebra in high school, geometry, and calculus, and it is true here too. If you use symbols—any symbols—make sure the meaning is clear to the reader *before* you use them.
- Think about your notation, and choose notation that is easy on the reader.
- A variable should only be assigned one meaning in your proof. For example, if you used C to denote the complex numbers, don't use C again to denote a different set.
- Try for a good blend of symbols and words. Don't juxtapose unrelated symbols if you don't have to. For example, consider the sentence "So $1 \leq p, q \geq 2$." You might find this confusing and (unnecessarily) difficult to read. If we say "So $1 \leq p$ and $q \geq 2$," the sentence is clear. It's often easier to read things if you put a word, even a little one, between symbols.
- Avoid starting a sentence with a symbol. This often confuses the reader unnecessarily. For example, consider the following sentence.

  Thus $x \in A. A$ is a subset of B.

  First, the $A.A$ just doesn't look nice. Second, it's hard to read.
- Every sentence should start with a capital letter and end with a period, just like sentences are supposed to begin and end.
- All grammatical rules apply. Make sure your sentence has a noun and a verb, for example.
- Strive for clarity. Always keep the reader in mind. If something follows from a definition, say so. The reader will appreciate this and will know what you are thinking *and*, what's more, you will know why what you say is true. If something follows from Theorem 10.1, say so. It is extremely important for you to be aware of when you are using a result. For one thing, it means that you are more likely to notice if you are using a result that you do not have. (This would be wrong. Don't do it.) For another, it helps the reader who may not fully understand what you are doing.

- Certain phrases are particularly helpful in guiding a reader through your proof. For example, "Suppose to the contrary, …" tells the reader that your proof will be done by contradiction. As a second example, if you are proving "A if and only if B," your reader will understand everything better if you say, "Suppose A. … Then we have B." And then say, "Suppose B. … Then we have A." You should alert the reader to a proof that will be in cases, or a proof that will proceed using the contrapositive. You should not only tell the reader how you will begin the proof, you should also tell the reader when you believe you have completed the proof. Words like "thus, we have established the desired result" will let the reader know that you think you are done now and it's his or her turn to understand why. Other examples of phrases that you may use to guide your reader will come up as we learn new techniques.
- If you can find a shorter, clearer solution, do so.
- Perhaps the most difficult thing about writing a proof is to find a balance between the main ideas in the proof and the details. You'll often find that the more you explain, the more you hide the main ideas. On the other hand, if you don't explain enough, you might overlook an important detail or confuse your reader. It's not easy to strike the right balance. This is why we suggest waiting a bit, and then rereading your proof. If you can't figure out why you did something, it's unlikely that someone else will.
- If you have a partner in the class, it is an excellent idea to exchange papers and see if things are clear to each of you. (Check with your teacher to make sure this is allowed, of course.)