问题与反馈

2015.4.9

6. In each of the following codes, what is the minimum distance for the code? What is the best situation we might hope for in connection with error detection and error correction?

   (a) (011010) (011100) (110111) (110000)

   (b) (011100) (011011) (111011) (100011)
   (000000) (010101) (110100) (110011)

   (c) (000000) (011100) (110101) (110001)

   (d) (0110110) (0111100) (1110000) (1111111)
   (1001001) (1000011) (0001111) (0000000)

Lemma 8.4 *Let* $\mathbf{x}$ *and* $\mathbf{y}$ *be binary* n*-tuples. Then* $w(\mathbf{x}+\mathbf{y}) = d(\mathbf{x},\mathbf{y})$.

Theorem 8.5 *Let* $d_{\min}$ *be the minimum distance for a group code* C*. Then* $d_{\min}$ *is the minimum of all the nonzero weights of the nonzero codewords in* C*. That is,*

$$d_{\min} = \min\{w(\mathbf{x}) : \mathbf{x} \neq \mathbf{0}\}.$$

9. Let C be the code obtained from the null space of the matrix

$$H = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

Decode the message

$$01111 \quad 10101 \quad 01110 \quad 00011$$

if possible.

11. Which matrices are canonical parity-check matrices? For those matrices that are canonical parity-check matrices, what are the corresponding standard generator matrices? What are the error-detection and error-correction capabilities of the code generated by each of these matrices?

(a)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

(d)

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Suppose that H is an $m \times n$ matrix with entries in $\mathbb{Z}_2$ and $n > m$. If the last m columns of the matrix form the $m \times m$ identity matrix, I_m, then the matrix is a ***canonical parity-check matrix***. More specifically, $H = (A \mid I_m)$, where A is the $m \times (n-m)$ matrix

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1,n-m} \\ a_{21} & a_{22} & \cdots & a_{2,n-m} \\ \vdots & \vdots \ddots & \vdots & \\ a_{m1} & a_{m2} & \cdots & a_{m,n-m} \end{pmatrix}$$

and I_m is the $m \times m$ identity matrix

$$\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots \ddots & \vdots & \\ 0 & 0 & \cdots & 1 \end{pmatrix}.$$

With each canonical parity-check matrix we can associate an $n \times (n-m)$ ***standard generator matrix***

$$G = \left(\frac{I_{n-m}}{A} \right).$$

Theorem 8.12 *Let H be an $m \times n$ binary matrix. Then the null space of H is a single error-detecting code if and only if no column of H consists entirely of zeros.*

Theorem 8.13 *Let H be a binary matrix. The null space of H is a single error-correcting code if and only if H does not contain any zero columns and no two columns of H are identical.*

21. If we are to use an error-correcting linear code to transmit the 128 ASCII characters, what size matrix must be used? What size matrix must be used to transmit the extended ASCII character set of 256 characters? What if we require only error detection in both cases?

23. How many check positions are needed for a single error-correcting code with 20 information positions? With 32 information positions?

Suppose now that we have a canonical parity-check matrix H with three rows. Then we might ask how many more columns we can add to the matrix and still have a null space that is a single error-detecting and single error-correcting code. Since each column has three entries, there are $2^3 = 8$ possible distinct columns. We cannot add the columns

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

So we can add as many as four columns and still maintain a minimum distance of 3.

In general, if H is an $m \times n$ canonical parity-check matrix, then there are $n - m$ information positions in each codeword. Each column has m bits, so there are 2^m possible distinct columns. It is necessary that the columns $\mathbf{0}, \mathbf{e}_1, \ldots, \mathbf{e}_n$ be excluded, leaving $2^m - (1 + n)$ remaining columns for information if we are still to maintain the ability not only to detect but also to correct single errors.

18. Let C be a linear code. Show that either the ith coordinates in the codewords of C are all zeros or exactly half of them are zeros.

19. Let C be a linear code. Show that either every codeword has even weight or exactly half of the codewords have even weight.

The End.