What We Talk About When We Talk About Isomorphism Theorems

Hengfeng Wei

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Theorem (The First Isomorphism Theorem)

$$\psi : G \to H \Longrightarrow \frac{G}{Ker\ \psi} \cong \psi(G)$$

Theorem (The Second Isomorphism Theorem)

$$H \leq G, N \lhd G \Longrightarrow \frac{H}{H \cap N} \cong \frac{HN}{N}$$

Theorem (The Third Isomorphism Theorem)

$$N \lhd H \lhd G \Longrightarrow \frac{G}{H} \cong \frac{G/N}{H/N}$$

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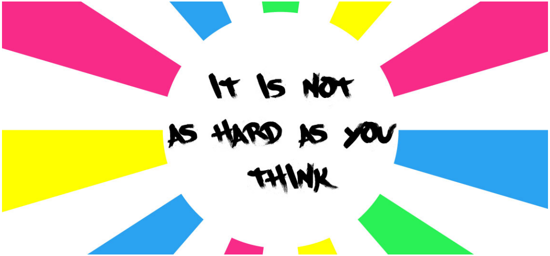
$$N \lhd G \Longrightarrow$$

$$\{\textit{(normal) subgroups of } G \textit{ containing } N\} \leftrightarrow \{\textit{(normal) subgroups of } G/N\}$$


IT IS NOT
AS HARD AS YOU
THINK

Q : Do isomorphic groups behave exactly the same?

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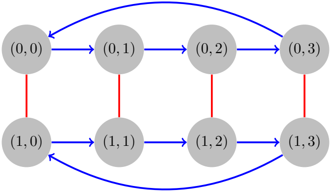

$K = \{(0,0),(0,2)\}$

$$G = Z_2 \times Z_4$$

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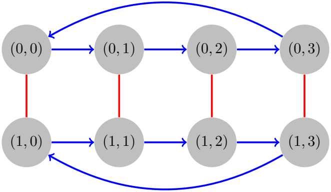

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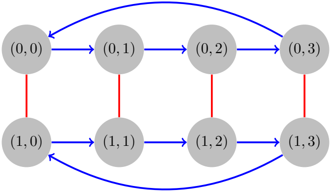

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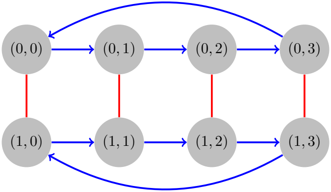

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Problem 9.3-23

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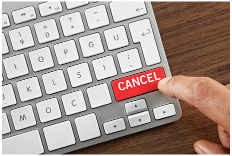

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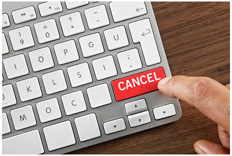

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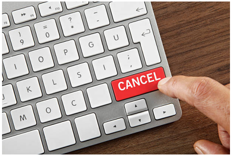

$$G = \mathbb{Z}, \quad H = \{e\}, \quad K = \prod_{n \in \mathbb{N}} \mathbb{Z}$$

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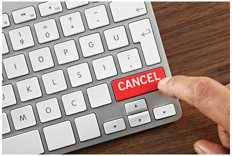

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"On Cancellation in Groups" by R. Hirshon, 1969

$$G \times K \cong H \times K, \ |K| < \infty \implies G \cong H$$

Problem 11.4-17

$\phi : G_1 \to G_2$ is a surjective group homomorphism.

$$H_1 \lhd G_1, \quad \phi(H_1) = H_2 \Longrightarrow G_1/H_1 \cong G_2/H_2$$

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$$G_1 = \mathbb{Z}_2, \quad G_2 = \{e\}, \quad H_1 = \{0\}, \quad H_2 = \{e\}$$

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Find all homomorphisms from $\mathbb{Z}_{24}$ to $\mathbb{Z}_{18}$.

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$$\phi(1) = 0, 9, 6, 12, 3, 15$$

Emmy Noether ($1882 \sim 1935$)

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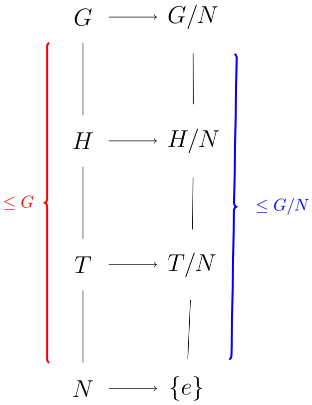

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Theorem (Ker ψ and Injectivity)

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Theorem (Ker ψ and Injectivity)

$$\psi : G \to H \ \textit{ is injective} \iff Ker\ \psi = \{e_G\}$$

$$\boxed{\frac{G}{\text{Ker } \psi} : \text{Quotient } G \text{ out by Ker } \psi}$$

$$\rho_1 = (2\ 3\ 4) \quad \rho_1^2 = (2\ 4\ 3)$$
$$\rho_2 = (1\ 3\ 4) \quad \rho_2^2 = (1\ 4\ 3)$$
$$\rho_3 = (1\ 2\ 4) \quad \rho_3^2 = (1\ 4\ 2)$$
$$\rho_4 = (1\ 2\ 3) \quad \rho_4^2 = (1\ 3\ 2)$$

$$r_1 = (1\ 4)(2\ 3)$$
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$$Sym(T) \cong A_4 = \left\{ \text{id}, \quad \underbrace{\text{3-cycle},}_{\#=8} \quad \underbrace{\text{2-2-cycle}}_{\#=3} \right\}$$

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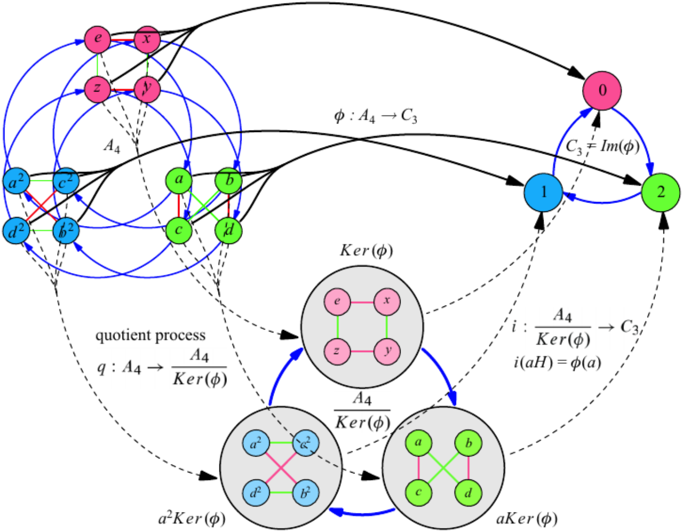
$$\frac{A_4}{\{1, r_1, r_2, r_3\}} \cong C_3$$



$$\phi : A_4 \to C_3 \qquad (\text{Ker } \phi = \{1, x, y, z\})$$

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$$\text{To show } \frac{G_1}{N} \cong G_2.$$

$$\frac{\mathbb{Z} \times \mathbb{Z}}{\langle (1,1) \rangle} \cong \mathbb{Z}$$

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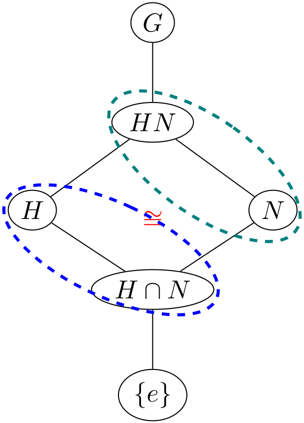

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$$h \in H \cap N \implies hN = N$$

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Problem 11.4-7

$$G = \mathbb{Z}_{24}, \quad H = \langle 4 \rangle, \quad N = \langle 6 \rangle$$

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$$ab = \gcd(a, b) \cdot \operatorname{lcm}(a, b)$$

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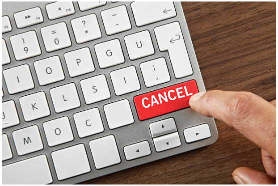

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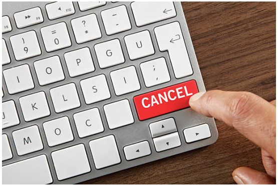

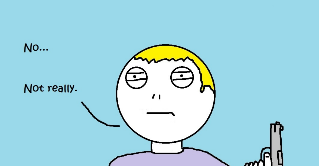

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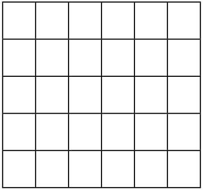

View G and H from the point of view of N

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Absorption!!!

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$$10\mathbb{Z} \lhd 2\mathbb{Z} \lhd \mathbb{Z}$$

$$\frac{\mathbb{Z}}{2\mathbb{Z}} \cong \frac{\mathbb{Z}/10\mathbb{Z}}{2\mathbb{Z}/10\mathbb{Z}}$$

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$$10\mathbb{Z} \lhd 2\mathbb{Z} \lhd \mathbb{Z}$$

$$\frac{\mathbb{Z}}{2\mathbb{Z}} \cong \frac{\mathbb{Z}/10\mathbb{Z}}{2\mathbb{Z}/10\mathbb{Z}}$$

$$\{0, 1\} \cong \frac{\{0, 1, 2, \cdots, 9\}}{}$$

Theorem (The Third Isomorphism Theorem)

$$N \lhd H \lhd G \Longrightarrow \frac{G}{H} \cong \frac{G/N}{H/N}$$

$$n \mid m$$

$$m\mathbb{Z} \lhd n\mathbb{Z} \lhd \mathbb{Z}$$

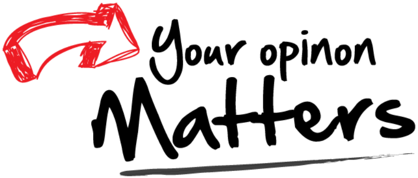
$$\frac{\mathbb{Z}}{n\mathbb{Z}} \cong \frac{\mathbb{Z}/m\mathbb{Z}}{n\mathbb{Z}/m\mathbb{Z}}$$

$$10\mathbb{Z} \lhd 2\mathbb{Z} \lhd \mathbb{Z}$$

$$\frac{\mathbb{Z}}{2\mathbb{Z}} \cong \frac{\mathbb{Z}/10\mathbb{Z}}{2\mathbb{Z}/10\mathbb{Z}}$$

$$\{0, 1\} \cong \frac{\{0, 1, 2, \cdots, 9\}}{\{0, 2, 4, 6, 8\}}$$


THANK YOU



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