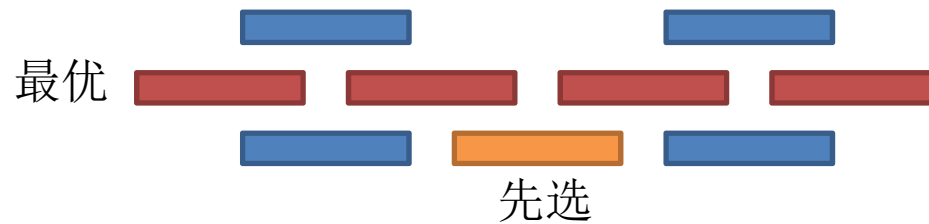


- 书面作业讲解
  - TC第16.1节练习2、3
  - TC第16.2节练习1、2
  - TC第16.3节练习2、5、8
  - TC第16章问题1
  - TC第17.1节练习3
  - TC第17.2节练习2
  - TC第17.4节练习1

# TC第16.1节练习3

- Overlaps the fewest other remaining activities.



# TC第16.2节练习1

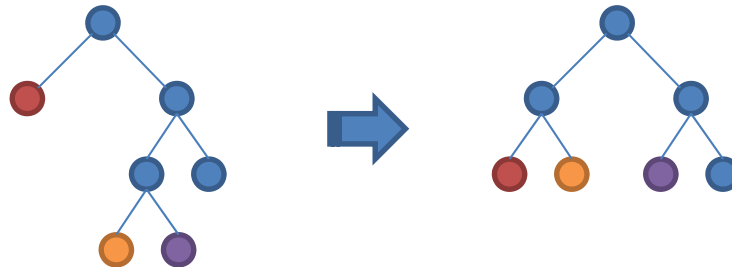
- 什么是greedy-choice property?
- 一般的证明方法是怎样的?
- 按性价比降序，如果第一个 $i$ 未被选入最优解，而是选了之后的某个 $j$ ，那么会怎样？
  - 若 $j$ 比 $i$ 的性价比低，替换之后可得更优解，与最优解矛盾
  - 若 $j$ 和 $i$ 的性价比相等，替换之后仍是最优解

# TC第16.3节练习5

- 这题并不限于Huffman编码，而是讨论任意的编码
- 频率： $F_1 \geq F_2 \geq \dots \geq F_n$
- 如果最优解不满足 $L_1 \leq L_2 \leq \dots \leq L_n$ ，即对于 $i < j$ ，有 $L_i > L_j$ ，那么交换 $i$ 和 $j$ ，新旧总长度之差为 $(F_j L_i + F_i L_j) - (F_i L_i + F_j L_j) = (F_j - F_i)(L_i - L_j) \leq 0$ 
  - 如果 $< 0$ ，与最优解矛盾
  - 如果 $= 0$ ，不断这样交换，可以调到 $L_1 \leq L_2 \leq \dots \leq L_n$

# TC第16.3节练习8

- 关于binary tree的用词
  - A **full** binary tree is a tree in which every node other than the leaves has two children.
  - A **complete** binary tree is a binary tree in which every level, except possibly the last, is completely filled, and all nodes are as far left as possible.
  - A **perfect** binary tree is a full binary tree in which all leaves are at the same depth or same level, and in which every parent has two children.
- 为什么这里Huffman编码树一定是perfect（深度都为8）？
  - 否则，必有一个深度 $<8$ ，两个深度 $>8$
  - 改变树形，提2降1，总长度减小，与最优解矛盾



# TC第16章问题1

- (b)
  - 如果greedy给出的不是最优解，那么最优解可以视作这种形式：
    - $c^k, \dots, c^{i+1}$ 都用满了（与greedy一致）
    - $c^i$ 没有用满（比greedy只少不多）
    - $c^{i-1}, \dots, c^0$ 先不管
  - 最优解需要用剩余的 $c^{i-1}, \dots, c^0$ 填补缺口：至少 $c^i$
  - 在填补中，如果每个 $c^j$ 都只用了少于 $c$ 个，最多只能填补 $c^i-1$ ，不合要求
  - 因此，必有一个 $c^j$ 用了不少于 $c$ 个，那么可以换成一个 $c^{j+1}$ ，与最优解矛盾

- 教材答疑和讨论  
– DW第1章

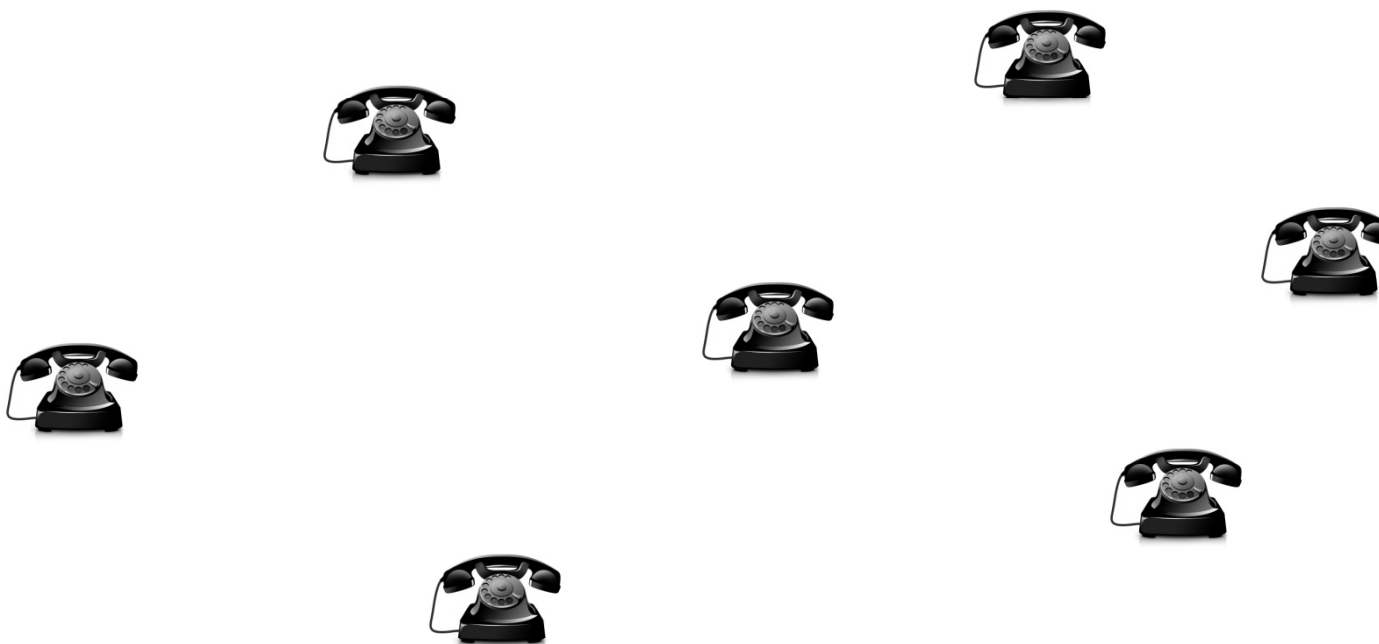


图论的学习，仿佛逻辑思维训练的“艺术体操”



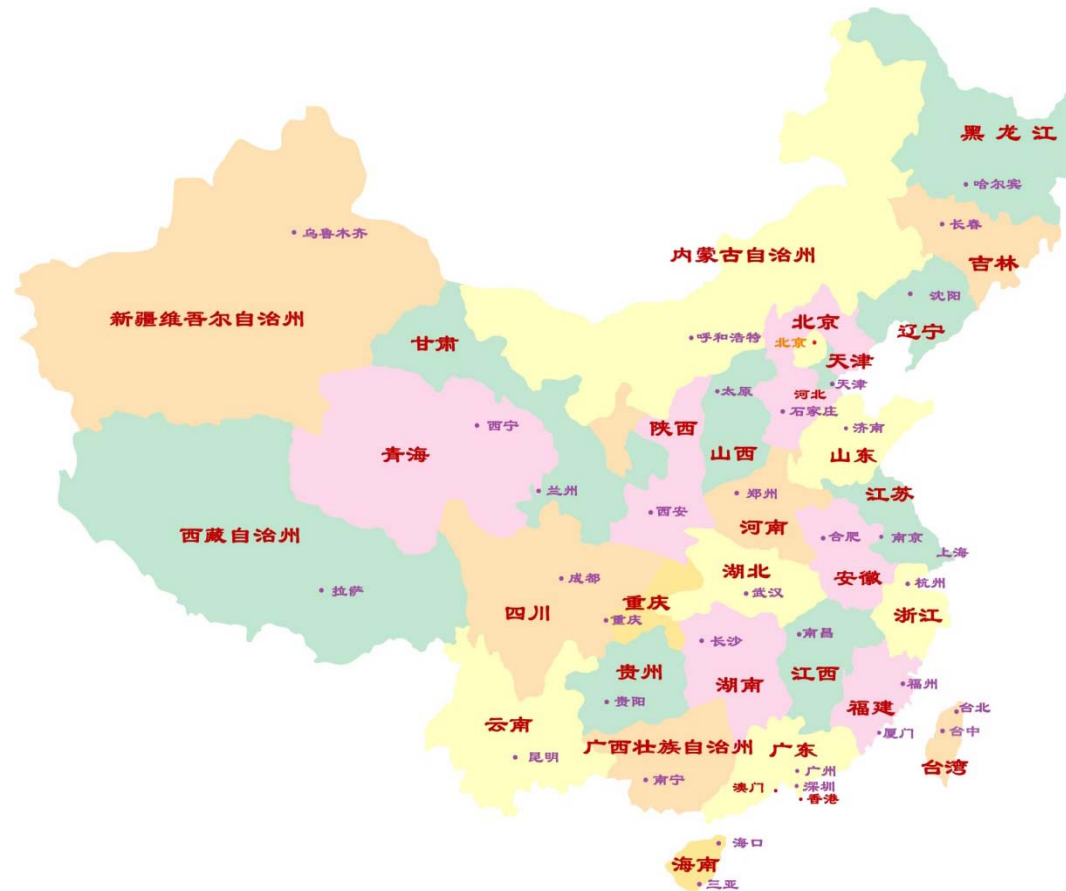
# 问题1：用图来建模问题

- How can we lay cable at minimum cost to make every telephone reachable from every other?



# 问题1: 用图来建模问题 (续)

- What is the fastest route from the national capital to each state capital?



# 问题1：用图来建模问题 (续)

- How can  $n$  jobs be filled by  $n$  people with maximum total utility?



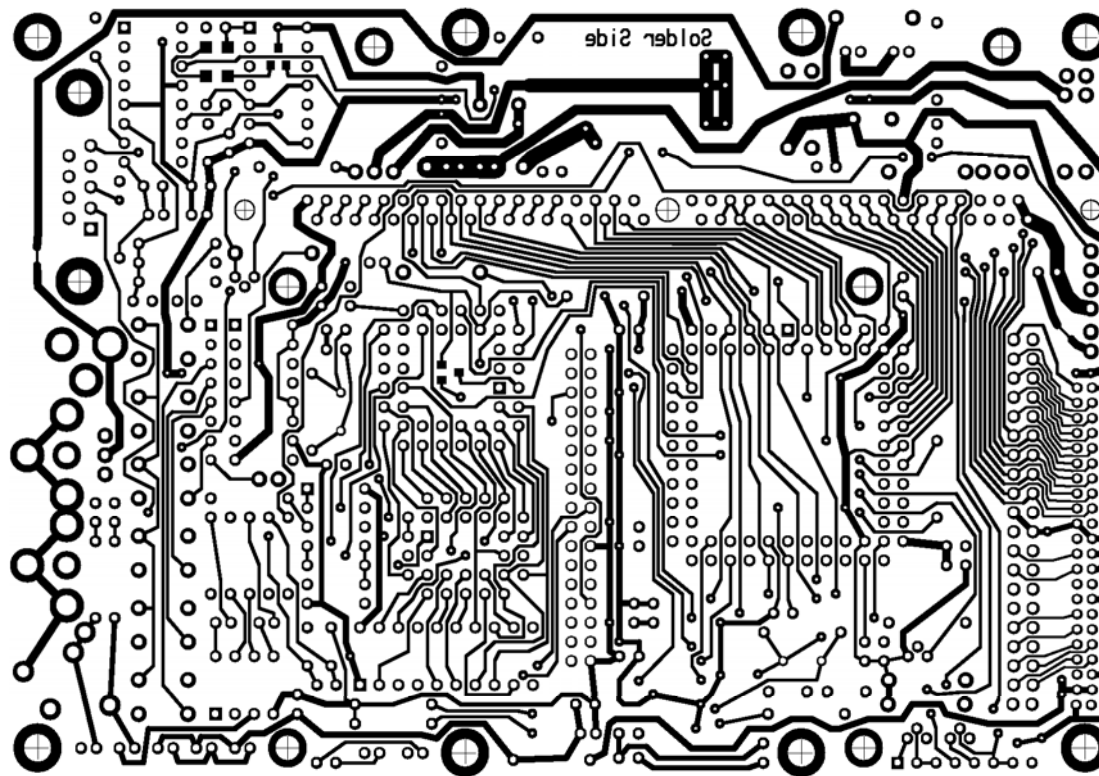
# 问题1：用图来建模问题 (续)

- What is the maximum flow per unit time from source to sink in a network of pipes?



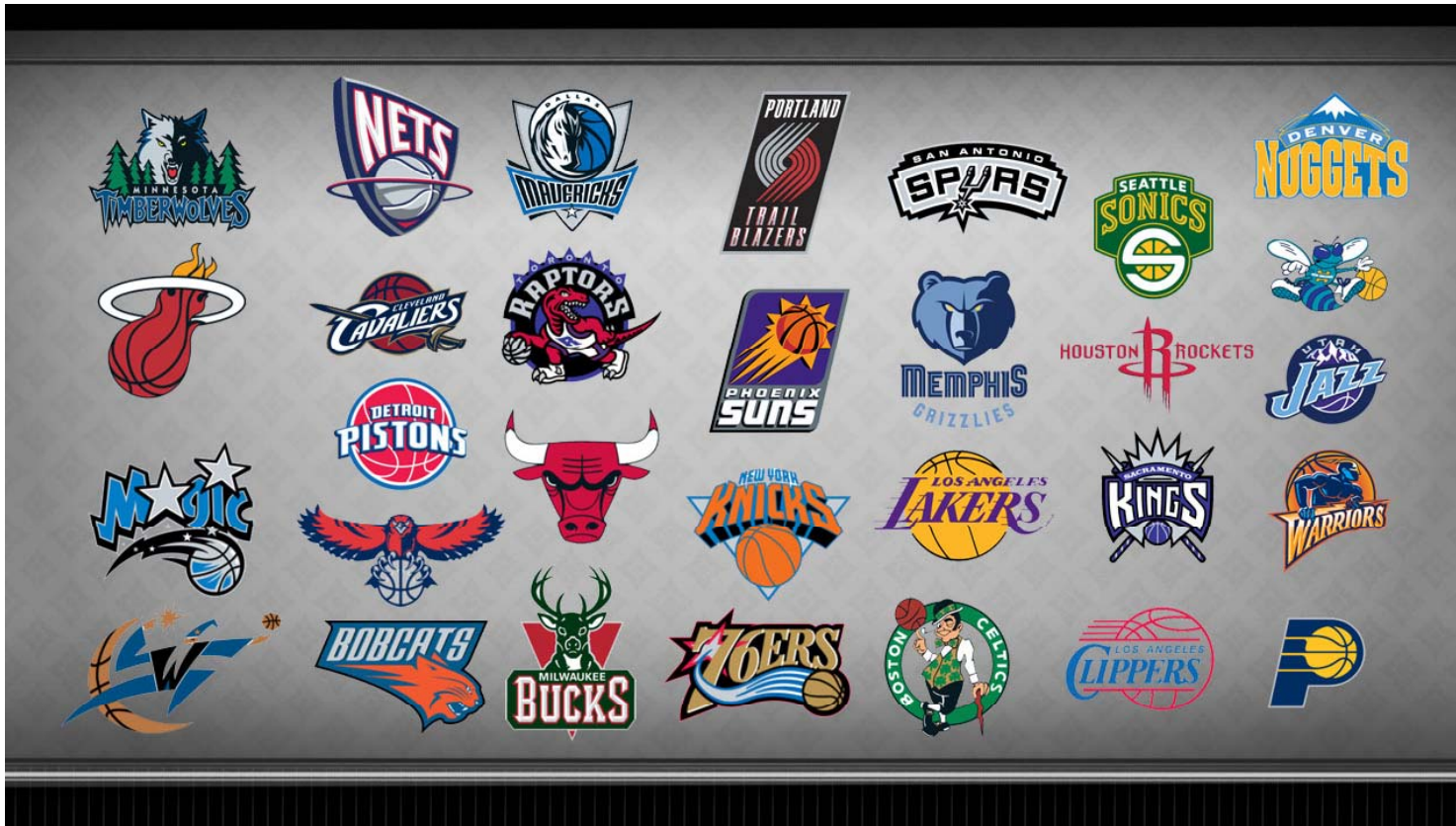
# 问题1: 用图来建模问题 (续)

- How many layers does a computer chip need so that wires in the same layer don't cross?



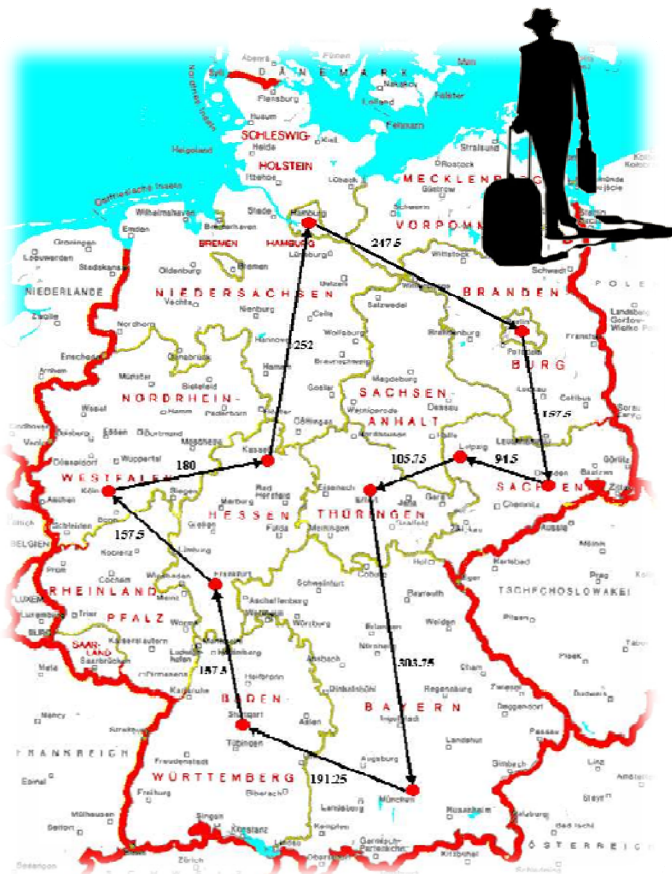
# 问题1: 用图来建模问题 (续)

- How can the season of a sports league be scheduled into the minimum number of weeks?



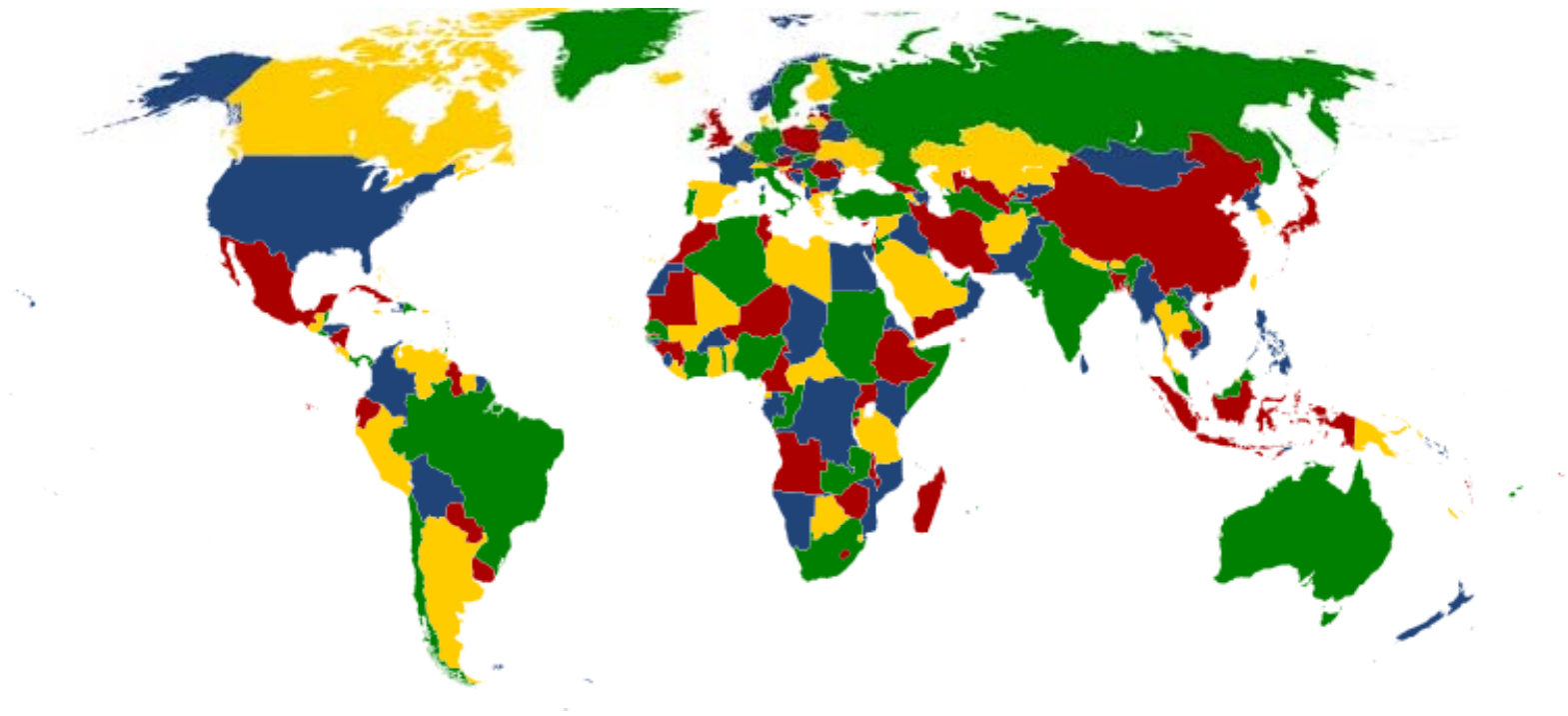
# 问题1: 用图来建模问题 (续)

- In what order should a traveling salesman visit cities to minimize travel time?



# 问题1: 用图来建模问题 (续)

- Can we color the regions of every map using four colors so that neighboring regions receive different colors?



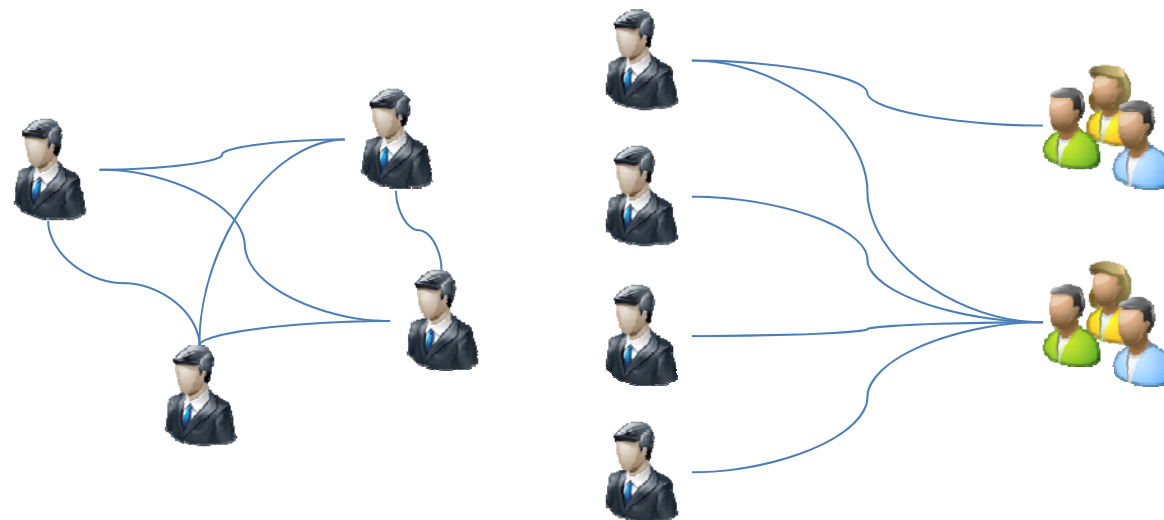


# 问题1：用图来建模问题 (续)

- the suspects
  - Severe acute respiratory syndrome (SARS), an atypical pneumonia of unknown aetiology, was recognized as a global threat in mid-March 2003. To minimize transmission to others, the best strategy is to separate the suspects from others.
  - In the Not-Spreading-Your-Sickness University (NSYSU), there are many student groups. Students in the same group intercommunicate with each other frequently, and a student may join several groups. To prevent the possible transmissions of SARS, the NSYSU collects the member lists of all student groups, and makes the following rule in their standard operation procedure (SOP).
    - Once a member in a group is a suspect, all members in the group are suspects.
    - However, they find that it is not easy to identify all the suspects when a student is recognized as a suspect. Your job is to write a program which finds all the suspects.

{A}

{A, B, C, D}



# 问题2：图的集合表示

- 如何用集合的语言来表示一个无向图？你能想到几种方式？

- 方法1

- $V=\{v1, v2, \dots\}$
- $E=\{e1, e2, \dots\}$
- $\text{endpoints}(e1)=\{v1, v2\}, \text{endpoints}(e2)=\{v3, v3\}, \dots$

- 方法2

- $V=\{v1, v2, \dots\}$
- $E=\{\{v1, v2\}, \{v3\}, \dots\}$  ←注意：这是一个multiset

- 它们各有什么优缺点？

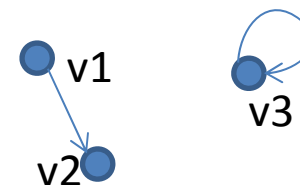
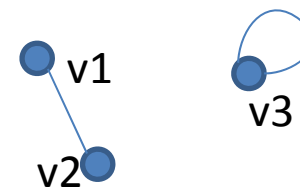
- 如何用集合的语言来表示一个有向图？

- 方法1

- $V=\{v1, v2, \dots\}$
- $E=\{e1, e2, \dots\}$
- $\text{tail}(e1)=v1, \text{tail}(e2)=v3, \dots$
- $\text{head}(e1)=v2, \text{head}(e2)=v3, \dots$

- 方法2

- $V=\{v1, v2, \dots\}$
- $E=\{\{v1\}, \{v1, v2\}, \{v3\}, \dots\}$



# 问题3：图论中的术语

- 环边 (loop)
- 重边 (multiple edges)
- 简单图 (simple graph)

## 问题3：图论中的术语 (续)

- 阶 (order)
- 边数 (size)
  - 边数的上下界是多少（与阶为单位）？
- 度 (degree)
  - 为什么奇度顶点的个数总是偶数？
- 孤立点 (isolated vertex)

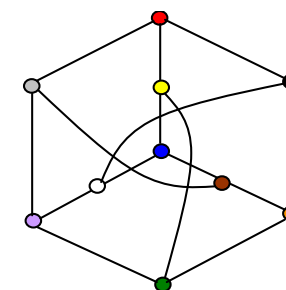
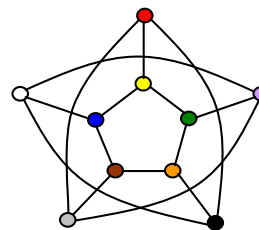
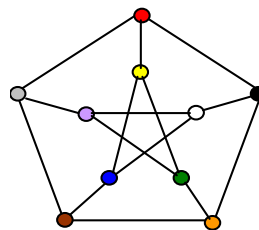
## 问题3：图论中的术语 (续)

- 零图 (null graph)
- 完全图 (complete graph)
- k-正则图 (k-regular graph)
  - 你能构造一个阶为5的3-正则图吗？
- 二部图 (bipartite graph)
  - 为什么不含奇圈是二部图的充要条件？
- 完全二部图 (complete bipartite graph)

# 问题3：图论中的术语 (续)

- 子图 (subgraph)
- 导出子图 (induced subgraph)
- 团 (clique)

- 同构 (isomorphism)
  - 你是怎么判断同构的?



- 补图 (complement)
  - 零图的补图是什么?
- 并 (union)
- 不交并/和 (disjoint union/sum)

# 问题3：图论中的术语 (续)

- 途径 (walk)
- 迹 (trail)
- 路/路径 (path)
  
- 闭途径 (closed walk)
- 闭迹 (closed trail)
- 圈 (cycle)
  - 为什么最小度为2的简单图必有圈？
  - 为什么最小度为3的简单图必有长度为偶数的圈？
  
- 连通 (connected)
- 连通分支/连通分量 (component)
  - 阶为 $n$ 的连通图最少有几条边？为什么？

# 问题3：图论中的术语 (续)

- 有向图 (digraph)
- 基础图/底图 (underlying graph)
- 定向图 (oriented graph)
  
- 入度 (indegree)
- 出度 (outdegree)
  
- 强连通 (strongly connected)
- 弱连通 (weakly connected)
- 强连通分支/强连通分量 (strong component)
  
- 竞赛图 (tournament)
  - 为什么出度为 $n-1$ 是唯一王的充要条件？