反馈与讨论

2014/4/17

16. An alternate version of the Ear Lemma states that a triangulated polygon is either a triangle with three ears or has at least two ears.

As an example, we will consider a theorem about triangulated polygons. To triangulate a polygon one keeps adding diagonals connecting pairs of vertices until no more diagonals can be added. These diagonals must lie entirely interior to the polygon and are not allowed to intersect. They break the interior of the polygon into a number of triangles, because any larger polygon can be split by adding a diagonal. (This fact is perhaps not obvious, but we won't get sidetracked by proving it here.) An example of a triangulated polygon appears in Figure 4.1. We say that a vertex of the polygon is incident to a diagonal if it is an endpoint of the diagonal. We define an ear of a triangulated polygon as a vertex that is not incident to any diagonal. We also say that two vertices are adjacent if they are connected by an edge in the polygon.

- The relationship between the number of vertices in a polygon and the number of triangles in any triangulation of that polygon. State this relation and prove it by induction.

9. Draw recursion trees, and use them to find big Θ bounds on the solutions to the following recurrences. For each, assume that $T(1) = 1$ and that n is a power of the appropriate integer.

a. $T(n) = 8T(n/2) + n$

b. $T(n) = 8T(n/2) + n^3$

c. $T(n) = 3T(n/2) + n$

d. $T(n) = T(n/4) + 1$

e. $T(n) = 3T(n/3) + n^2$

16. If $S(n) = aS(n-1) + g(n)$ and $g(n) < c^n$ with $1 \leq c < a$, how fast does $S(n)$ grow (in big Θ terms)?

17. If $S(n) = aS(n-1) + g(n)$ and $g(n) = c^n$ with $0 < a < c$, how fast does $S(n)$ grow in big Θ terms?

4. Give a big Θ bound on the solution to the recurrence

$$T(n) = \begin{cases} 3T(\lceil n/2 \rceil) + \sqrt{n^4 + 3} & \text{if } n > 1, \\ d & \text{if } n = 1. \end{cases}$$

10. Give the best big O upper bound you can for the solution to the recurrence

$$T(n) = 2T\left(\frac{n}{3} - 3\right) + n$$

(making an informed guess is not a bad idea here). Then prove by induction that your upper bound is correct.

Collatz conjecture

- (1) 如果n等于1，exit;
- (2) 如果n是偶数,n←n/2;
- (3) 如果n是奇数，n←3n+1;
- Goto (1)
- Collatz猜想是说上述过程一定会终止，就是任何一个正整数通过上述过程都可以归一。
- 尝试用递归和非递归程序实现。