计算机问题求解 – 论题2–12

- 动态规划

课程研讨

- TC第15章

问题1：dynamic programming的基本概念

- 什么样的问题可以使用dynamic programming来求解？它高效的根本原因是什么？付出了什么代价？
- 你理解dynamic programming的四个步骤了吗？
  1. Characterize the structure of an optimal solution.
  2. Recursively define the value of an optimal solution.
  3. Compute the value of an optimal solution.
  4. Construct an optimal solution from computed information.

- 广义上决定dynamic programming运行时间的要素是哪两点？
- top-down with memorization和bottom-up method哪个更快？

问题2：dynamic programming的实例

- 你能说明求解rod cutting的四个步骤吗？
  1. Characterize the structure of an optimal solution.
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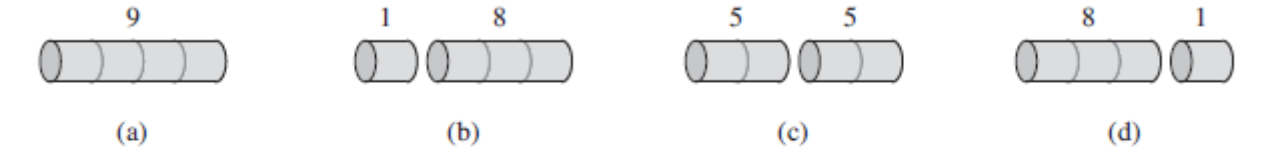

length i	1	2	3	4	5	6	7	8	9	10
price p_i	1	5	8	9	10	17	17	20	24	30

问题2：dynamic programming的实例

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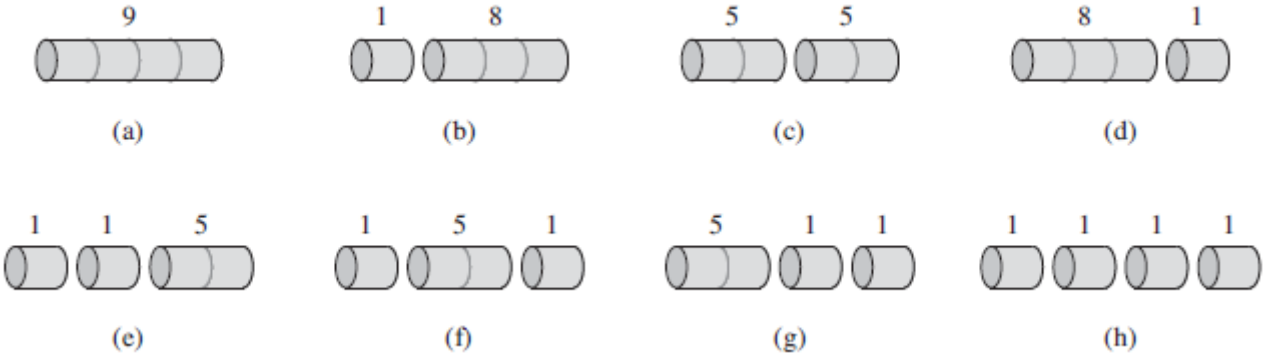
$$r_n = \max_{1 \le i \le n} (p_i + r_{n-i})$$

```
PRINT-CUT-ROD-SOLUTION(p, n)
1  (r, s) = EXTENDED-BOTTOM-UP-CUT-ROD(p, n)
2  while n > 0
3      print s[n]
4      n = n − s[n]
```



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问题2：dynamic programming的实例 (续)

- 你能说明求解matrix-chain multiplication的四个步骤吗？
  1. Characterize the structure of an optimal solution.
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$$
\begin{aligned}
&(A_1(A_2(A_3A_4)))\,,\\
&(A_1((A_2A_3)A_4))\,,\\
&((A_1A_2)(A_3A_4))\,,\\
&((A_1(A_2A_3))A_4)\,,\\
&(((A_1A_2)A_3)A_4)\,.
\end{aligned}
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$$m[i,j] = \begin{cases} 0 & \text{if } i = j\ , \\ \min\limits_{i \le k < j} \{m[i,k] + m[k+1,j] + p_{i-1} p_k p_j\} & \text{if } i < j\ . \end{cases}$$

```
PRINT-OPTIMAL-PARENS (s, i, j)
1  if i == j
2      print "A"_i
3  else print "("
4      PRINT-OPTIMAL-PARENS (s, i, s[i, j])
5      PRINT-OPTIMAL-PARENS (s, s[i, j] + 1, j)
6      print ")"
```

$$\begin{aligned} &(A_1(A_2(A_3A_4)))\ , \\ &(A_1((A_2A_3)A_4))\ , \\ &((A_1A_2)(A_3A_4))\ , \\ &((A_1(A_2A_3))A_4)\ , \\ &(((A_1A_2)A_3)A_4)\ . \end{aligned}$$

问题2：dynamic programming的实例 (续)

- 你能说明求解optimal binary search trees的四个步骤吗？
  1. Characterize the structure of an optimal solution.
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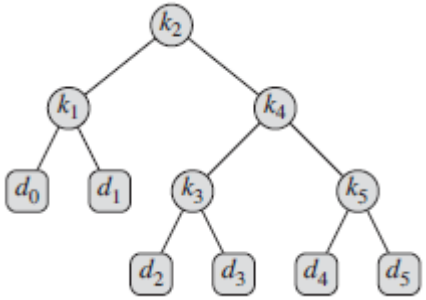

i	0	1	2	3	4	5
p_i		0.15	0.10	0.05	0.10	0.20
q_i	0.05	0.10	0.05	0.05	0.05	0.10

问题2：dynamic programming的实例 (续)

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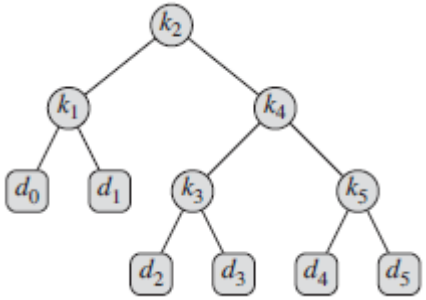
$$e[i,j] = \begin{cases} q_{i-1} & \text{if } j = i-1, \\ \min\limits_{i \le r \le j} \{e[i,r-1] + e[r+1,j] + w(i,j)\} & \text{if } i \le j. \end{cases}$$



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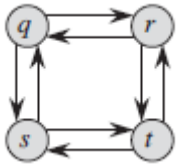
问题2：dynamic programming的实例 (续)

- unweighted longest simple path为什么不具有最优子结构？
- unweighted shortest simple path为什么不存在这个问题？



问题2：dynamic programming的实例 (续)

- 你能说明求解longest common subsequence的四个步骤吗？
  1. Characterize the structure of an optimal solution.
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S_1 = ACCGGTCGAGTGCGCGGAAGCCGGCCGAA

S_2 = GTCGTTCGGAATGCCGTTGCTCTGTAAA

GTCGTCGGAAGCCGGCCGAA

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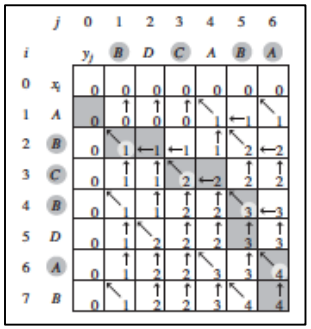
$$c[i,j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0, \\ c[i-1,j-1]+1 & \text{if } i,j > 0 \text{ and } x_i = y_j, \\ \max(c[i,j-1], c[i-1,j]) & \text{if } i,j > 0 \text{ and } x_i \neq y_j. \end{cases}$$



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问题2：dynamic programming的实例 (续)

- 你能说明求解longest palindrome subsequence的四个步骤吗？
  1. Characterize the structure of an optimal solution.
  2. Recursively define the value of an optimal solution.
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A *palindrome* is a nonempty string over some alphabet that reads the same forward and backward. Examples of palindromes are all strings of length 1, `civic`, `racecar`, and `aibohphobia` (fear of palindromes).

Give an efficient algorithm to find the longest palindrome that is a subsequence of a given input string. For example, given the input `character`, your algorithm should return `carac`.

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```
longest(i,j)= j-i+1 if j-i<=0,
              2+longest(i+1,j-1) if x[i]==x[j]
              max(longest(i+1,j),longest(i,j-1)) otherwise
```

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问题2：dynamic programming的实例 (续)

- 你能说明求解edit distance的四个步骤吗？
  1. Characterize the structure of an optimal solution.
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Insertion of a single symbol. If $a = uv$, then inserting the symbol x produces uxv. This can also be denoted $\varepsilon \to x$, using ε to denote the empty string.

Deletion of a single symbol changes uxv to uv $(x \to \varepsilon)$.

Substitution of a single symbol x for a symbol $y \neq x$ changes uxv to uyv $(x \to y)$.

The Levenshtein distance between "kitten" and "sitting" is 3. The minimal edit script that transforms the former into the latter is:

1. **k**itten → **s**itten (substitution of "s" for "k")
2. sitt**e**n → sitt**i**n (substitution of "i" for "e")
3. sittin → sitting (insertion of "g" at the end).

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$$d_{ij} = \begin{cases} d_{i-1,j-1} & \text{for } a_j = b_i \\ \min \begin{cases} d_{i-1,j} + w_{\text{del}}(b_i) \\ d_{i,j-1} + w_{\text{ins}}(a_j) \\ d_{i-1,j-1} + w_{\text{sub}}(a_j, b_i) \end{cases} & \text{for } a_j \neq b_i \end{cases} \qquad \text{for } 1 \leq i \leq m, 1 \leq j \leq n.$$

Insertion of a single symbol. If $a = uv$, then inserting the symbol x produces uxv. This can also be denoted $\varepsilon \to x$, using ε to denote the empty string.

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Separating Sequence of Words

- Word-length $w_1, w_2, \ldots, w_n$ and line-width: W
- Basic constraint: if $w_i, w_{i+1}, \ldots, w_j$ are in one line, then $w_i+w_{i+1}+ \ldots+w_j \leq W$
- Penalty for one line: some function of X. X is:
  - 0 for the last line in a paragraph, and
  - $W -(w_i+w_{i+1}+ \ldots+w_j)$ for other lines
- The problem
  - how to separate a sequence of words(forming a paragraph) into lines, making the penalty of the paragraph, which is the sum of the penalties of individual lines, minimized.

Solution by Greedy Strategy

i	word	*w*
1	Those	6
2	who	4
3	cannot	7
4	remember	9
5	the	4
6	past	5
7	are	4
8	condemned	10
9	to	3
10	repeat	7
11	it.	4

W is 17, and penalty is X^3

Solution by greedy strategy

words	(1,2,3)	(4,5)	(6,7)	(8,9)	(10,11)
X	0	4	8	4	0
penalty	0	64	512	64	0

Total penalty is **640**

An improved solution

words	(1,2)	(3,4)	(5,6,7)	(8,9)	(10,11)
X	7	1	4	4	0
penalty	343	1	64	64	0

Total penalty is **472**