问题与反馈

2015/1/5

29.2-3

In the single-source shortest-paths problem, we want to find the shortest-path weights from a source vertex s to all vertices $v \in V$. Given a graph G, write a linear program for which the solution has the property that d_v is the shortest-path weight from s to v for each vertex $v \in V$.

29.2-6

Write a linear program that, given a bipartite graph $G = (V, E)$, solves the maximum-bipartite-matching problem.

29.3-2

Show that the call to PIVOT in line 12 of SIMPLEX never decreases the value of ν.

```
SIMPLEX(A, b, c)
1  (N, B, A, b, c, ν) = INITIALIZE-SIMPLEX(A, b, c)
2  let Δ be a new vector of length n
3  while some index j ∈ N has c_j > 0
4      choose an index e ∈ N for which c_e > 0
5      for each index i ∈ B
6          if a_ie > 0
7              Δ_i = b_i / a_ie
8          else Δ_i = ∞
9      choose an index l ∈ B that minimizes Δ_i
10     if Δ_l == ∞
11         return "unbounded"
12     else (N, B, A, b, c, ν) = PIVOT(N, B, A, b, c, ν, l, e)
13 for i = 1 to n
14     if i ∈ B
15         x̄_i = b_i
16     else x̄_i = 0
17 return (x̄_1, x̄_2, ..., x̄_n)
```

```
PIVOT(N, B, A, b, c, ν, l, e)
// Compute the coefficients of the equation for new basic variable x_e.
let Â be a new m × n matrix
b̂_e = b_l / a_le
for each j ∈ N − {e}
    â_ej = a_lj / a_le
â_el = 1 / a_le
// Compute the coefficients of the remaining constraints.
for each i ∈ B − {l}
    b̂_i = b_i − a_ie b̂_e
    for each j ∈ N − {e}
        â_ij = a_ij − a_ie â_ej
    â_il = −a_ie â_el
// Compute the objective function.
ν̂ = ν + c_e b̂_e
for each j ∈ N − {e}
    ĉ_j = c_j − c_e â_ej
ĉ_l = −c_e â_el
// Compute new sets of basic and nonbasic variables.
N̂ = N − {e} ∪ {l}
B̂ = B − {l} ∪ {e}
return (N̂, B̂, Â, b̂, ĉ, ν̂)
```

29.3-5

Solve the following linear program using SIMPLEX:

$$
\begin{array}{lrcrcr}
\text{maximize} & 18x_1 & + & 12.5x_2 & & \\
\text{subject to} & & & & & \\
 & x_1 & + & x_2 & \leq & 20 \\
 & x_1 & & & \leq & 12 \\
 & & & x_2 & \leq & 16 \\
 & & x_1, x_2 & & \geq & 0 .
\end{array}
$$

29-1 Linear-inequality feasibility

Given a set of m linear inequalities on n variables $x_1, x_2, \ldots, x_n$, the ***linear-inequality feasibility problem*** asks whether there is a setting of the variables that simultaneously satisfies each of the inequalities.

a. Show that if we have an algorithm for linear programming, we can use it to solve a linear-inequality feasibility problem. The number of variables and constraints that you use in the linear-programming problem should be polynomial in n and m.

b. Show that if we have an algorithm for the linear-inequality feasibility problem, we can use it to solve a linear-programming problem. The number of variables and linear inequalities that you use in the linear-inequality feasibility problem should be polynomial in n and m, the number of variables and constraints in the linear program.

$$\begin{aligned}
\text{maximize} \quad & -x_0 && (29.106)\\
\text{subject to} \quad & && \\
& \sum_{j=1}^{n} a_{ij}x_j - x_0 \leq b_i \quad \text{for } i = 1, 2, \ldots, m\,, && (29.107)\\
& x_j \geq 0 \quad \text{for } j = 0, 1, \ldots, n\,. && (29.108)
\end{aligned}$$

Then L is feasible if and only if the optimal objective value of L_{aux} is 0.

$$\begin{aligned}
\text{maximize} \quad & \sum_{j=1}^{n} c_j x_j && (29.16)\\
\text{subject to} \quad & && \\
& \sum_{j=1}^{n} a_{ij}x_j \leq b_i \quad \text{for } i = 1, 2, \ldots, m && (29.17)\\
& x_j \geq 0 \quad \text{for } j = 1, 2, \ldots, n\,. && (29.18)
\end{aligned}$$

$$\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{m} b_i y_i && (29.83)\\
\text{subject to} \quad & && \\
& \sum_{i=1}^{m} a_{ij} y_i \geq c_j \quad \text{for } j = 1, 2, \ldots, n\,, && (29.84)\\
& y_i \geq 0 \quad \text{for } i = 1, 2, \ldots, m\,. && (29.85)
\end{aligned}$$