计算机问题求解 – 论题2-13

- 贪心算法

课程研讨

- TC第16.1-16.3节、第17章

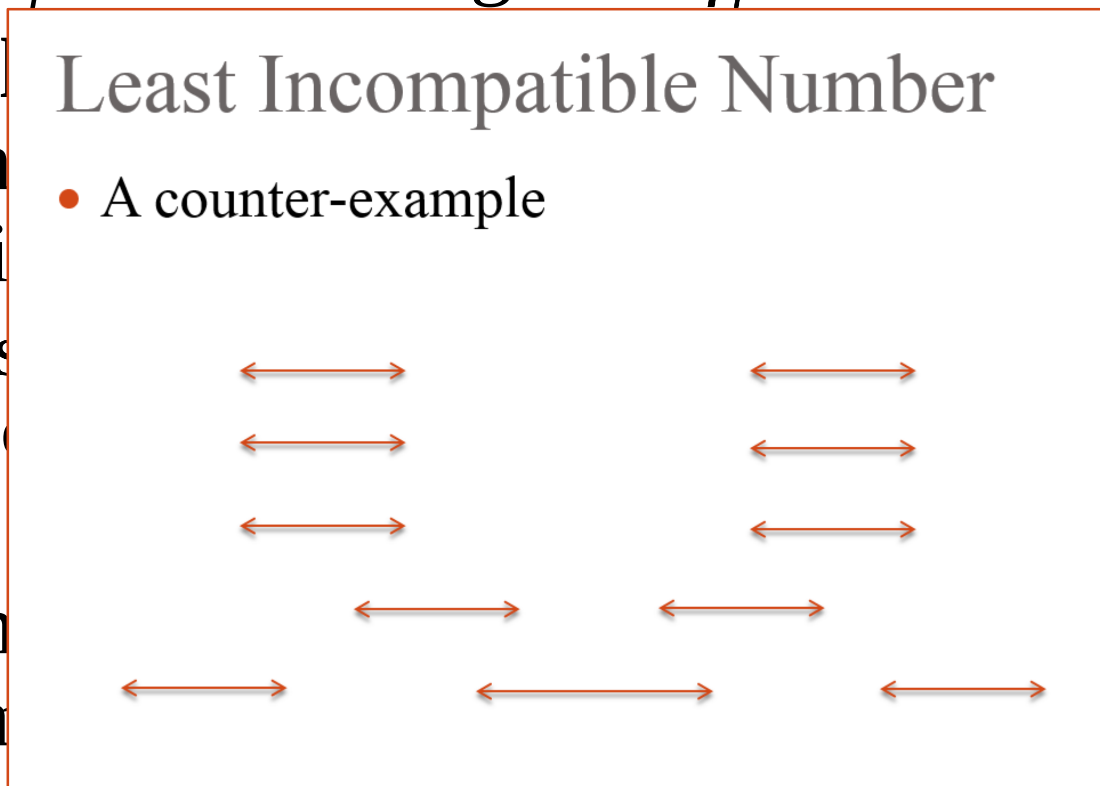
问题1： greedy algorithms

- 你怎么理解greedy algorithms的两个重要性质？
  - greedy-choice property
  - optimal substructure
- 你能不能结合activity-selection problem解释为什么这两个性质缺一不可？
- 为什么greedy algorithms比dynamic programming快？

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

Scheduling Activities

- **Instance**: a set of *n* activities, each with start time s_i and finishing time f_i
- **Probl** … with the m … ping activi …
  - We s … they do **not** …
- **Idea**: … finish … are incon … then repeat…



A Scheduling Algorithm

- Sort activities by finish time.
- Choose first activity.
- Repeatedly choose the next activity that is compatible with all previously chosen ones.

- Running time: $\Theta(n \log n)$ time to sort, $\Theta(n)$ time for the rest.
- How do we prove this is correct?

问题1： greedy algorithms

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- 为什么greedy algorithms比dynamic programming快？
  - making the first choice before solving any subproblems
  - making one greedy choice after another
    reducing each given problem instance to a smaller one

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s_i	1	3	0	5	3	5	6	8	8	2	12
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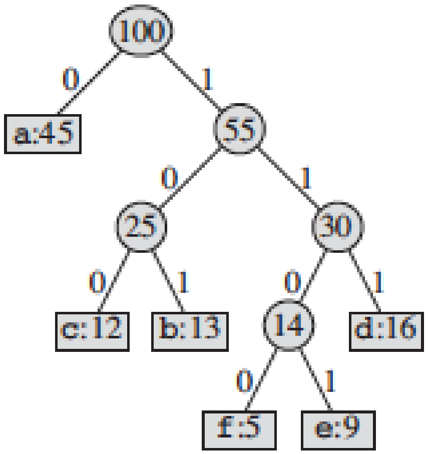
问题1： greedy algorithms (续)

- 关于Huffman codes的greedy algorithm
  - greedy choice是什么？
  - greedy-choice property是什么？
  - optimal substructure是什么？

	a	b	c	d	e	f
Frequency (in thousands)	45	13	12	16	9	5
Fixed-length codeword	000	001	010	011	100	101
Variable-length codeword	0	101	100	111	1101	1100



问题1： greedy algorithms (续)

Describe an efficient algorithm that, given a set $\{x_1, x_2, \ldots, x_n\}$ of points on the real line, determines the smallest set of unit-length closed intervals that contains all of the given points. Argue that your algorithm is correct.

问题1： greedy algorithms (续)

Describe an efficient algorithm that, given a set $\{x_1, x_2, \ldots, x_n\}$ of points on the real line, determines the smallest set of unit-length closed intervals that contains all of the given points. Argue that your algorithm is correct.

Sol: First we sort the set of n points $\{x_1, x_2, ..., x_n\}$ to get the set $Y = \{y_1, y_2, ..., y_n\}$ such that $y_1 \le y_2 \le ... \le y_n$. Next, we do a linear scan on $\{y_1, y_2, ..., y_n\}$ started from y_1. Everytime while encountering y_i, for some $i \in \{1, ..., n\}$, we put the closed interval $[y_i, y_i + 1]$ in our optimal solution set S, and remove all the points in Y covered by $[y_i, y_i + 1]$. Repeat the above procedure, finally output S while Y becomes empty. We next show that S is an optimal solution.

We claim that there is an optimal solution which contains the unit-length interval $[y_1, y_1 + 1]$. Suppose that there exists an optimal solution S^* such that y_1 is covered by $[x', x'+1] \in S^*$ where $x' < 1$. Since y_1 is the leftmost element of the given set, there is no other point lying in $[x', y_1)$. Therefore, if we replace $[x', x'+1]$ in S^* by $[y_1, y_1+1]$, we will get another optimal solution. This proves the claim and thus explains the greedy choice property. Therefore, by solving the remaining subproblem after removing all the points lying in $[y_1, y_1 + 1]$, that is, to find an optimal set of intervals, denoted as S', which cover the points to the right of $y1 + 1$, we will get an optimal solution to the original problem by taking union of $[y_1, y_1 + 1]$ and S'.

Scheduling to Minimize Lateness

- **Instance**: a set of *n* activities, each with start time s_i, deadline d_i and a duration t_i.
- **Problem**: we plan to satisfy each request, but we are allowed to let certain requests run late, and the optimization goal is to schedule all requests, using non-overlapping intervals, so as to minimize the ma
  - We say a request *i* is
    and the lateness of s
    $l_i=f(i)-d_i$.
  - The goal: minimize

Greedy Strategies

- Choosing the smallest t_i
- Choosing the smallest (d_i-t_i)
- Choosing the smallest d_i

问题1： greedy algorithms (续)

- 建议阅读打星号的16.4节

问题2： amortized analysis

- amortized analysis和average-case analysis有什么异同？

问题2： amortized analysis

- amortized analysis和average-case analysis有什么异同？
  - per operation vs. per algorithm
  - worst-case vs. average-case

问题2： amortized analysis (续)

- 这些问题的分析难在哪儿？
amortized analysis能带来什么好处？
  - stack operations

PUSH(S, x) pushes object x onto stack S.

POP(S) pops the top of stack S and returns the popped object. Calling POP on an empty stack generates an error.

```
MULTIPOP(S, k)
while not STACK-EMPTY(S) and k > 0
    POP(S)
    k = k - 1
```

  - incrementing a binary counter

Counter value	A[7]	A[6]	A[5]	A[4]	A[3]	A[2]	A[1]	A[0]	Total cost
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	1	1
2	0	0	0	0	0	0	1	0	3
3	0	0	0	0	0	0	1	1	4
4	0	0	0	0	0	1	0	0	7
5	0	0	0	0	0	1	0	1	8
6	0	0	0	0	0	1	1	0	10
7	0	0	0	0	0	1	1	1	11
8	0	0	0	0	1	0	0	0	15
9	0	0	0	0	1	0	0	1	16
10	0	0	0	0	1	0	1	0	18
11	0	0	0	0	1	0	1	1	19
12	0	0	0	0	1	1	0	0	22
13	0	0	0	0	1	1	0	1	23
14	0	0	0	0	1	1	1	0	25
15	0	0	0	0	1	1	1	1	26
16	0	0	0	1	0	0	0	0	31

问题2： amortized analysis (续)

- aggregate analysis
  - 该方法的基本思路是什么？
  - 如何用来解决这两个问题？
    - 如果增加一个DECREMENT，结果又如何？
  - 它在使用上有什么局限吗？

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0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	1	1
2	0	0	0	0	0	0	1	0	3
3	0	0	0	0	0	0	1	1	4
4	0	0	0	0	0	1	0	0	7
5	0	0	0	0	0	1	0	1	8
6	0	0	0	0	0	1	1	0	10
7	0	0	0	0	0	1	1	1	11
8	0	0	0	0	1	0	0	0	15
9	0	0	0	0	1	0	0	1	16
10	0	0	0	0	1	0	1	0	18
11	0	0	0	0	1	0	1	1	19
12	0	0	0	0	1	1	0	0	22
13	0	0	0	0	1	1	0	1	23
14	0	0	0	0	1	1	1	0	25
15	0	0	0	0	1	1	1	1	26
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问题2： amortized analysis (续)

- accounting method
  - 该方法的基本思路是什么？
    右侧这个式子是什么含义？ $$\sum_{i=1}^{n} \hat{c}_i \geq \sum_{i=1}^{n} c_i$$
  - 如何用来解决这两个问题？
    上述式子是如何保证成立的？
    - 如果增加一个RESET，结果又如何？
      (Keep a pointer to the high-order 1.)

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1	0	0	0	0	0	0	0	1	1
2	0	0	0	0	0	0	1	0	3
3	0	0	0	0	0	0	1	1	4
4	0	0	0	0	0	1	0	0	7
5	0	0	0	0	0	1	0	1	8
6	0	0	0	0	0	1	1	0	10
7	0	0	0	0	0	1	1	1	11
8	0	0	0	0	1	0	0	0	15
9	0	0	0	0	1	0	0	1	16
10	0	0	0	0	1	0	1	0	18
11	0	0	0	0	1	0	1	1	19
12	0	0	0	0	1	1	0	0	22
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问题2： amortized analysis (续)

- potential method
  - 该方法的基本思路是什么？右侧这组式子是什么含义？
  - 如何用来解决这两个问题？以及一个新问题：

    Implement a queue with two stacks.
    The amortized cost of ENQ and DEQ is O(1).

$$\hat{c}_i = c_i + \Phi(D_i) - \Phi(D_{i-1}) .$$

$$\sum_{i=1}^{n} \hat{c}_i = \sum_{i=1}^{n} (c_i + \Phi(D_i) - \Phi(D_{i-1})) = \sum_{i=1}^{n} c_i + \Phi(D_n) - \Phi(D_0) .$$

$$\Phi(D_n) \geq \Phi(D_0)$$

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1	0	0	0	0	0	0	0	1	1
2	0	0	0	0	0	0	1	0	3
3	0	0	0	0	0	0	1	1	4
4	0	0	0	0	0	1	0	0	7
5	0	0	0	0	0	1	0	1	8
6	0	0	0	0	0	1	1	0	10
7	0	0	0	0	0	1	1	1	11
8	0	0	0	0	1	0	0	0	15
9	0	0	0	0	1	0	0	1	16
10	0	0	0	0	1	0	1	0	18
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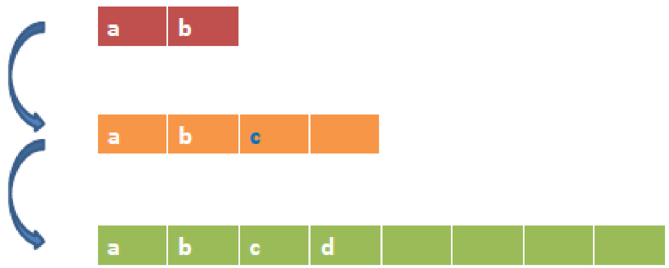
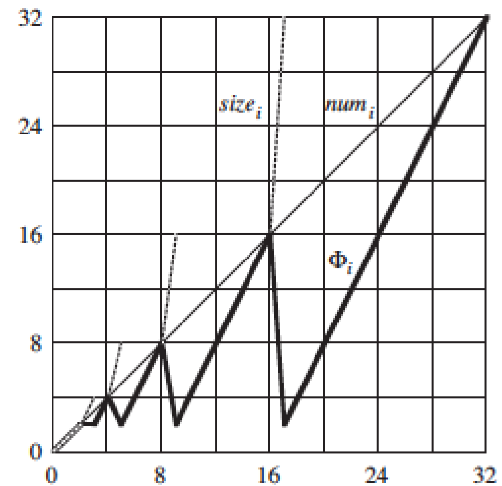
问题3： dynamic tables

- 对于table expansion， 你能解释aggregate和accounting的分析过程吗？

$$c_i = \begin{cases} i & \text{if } i-1 \text{ is an exact power of 2 ,} \\ 1 & \text{otherwise .} \end{cases}$$

$$\begin{aligned} \sum_{i=1}^{n} c_i &\le n + \sum_{j=0}^{\lfloor \lg n \rfloor} 2^j \\ &< n + 2n \\ &= 3n , \end{aligned}$$





- 对于potential function $\Phi(T) = 2 \cdot T.num - T.size$
你能结合accounting来解释它的走势吗？

Graph Operations

- Consider the following operations on a set of nodes in a graph:
  - Connect(A, B): add an edge from node A to node B in the graph (if there already exists such an edge, do nothing);
  - Disconnect(A, B): if there are paths from A to B, remove all edges in the paths(if there is no path, do nothing);
- Assume the cost of adding an edge is 1, removing an edge is 2, and the cost of finding a path in the graph is omitted. There is no edge in the graph at the beginning. Consider a sequence of *n* operations on the graph, apply amortized analysis on the cost of Connect and Disconnect operations in the worst case.

Implement a Queue with two Stacks

- Describe how to implement a queue with two stacks which are implemented by arrays. Analyze the complexity of *Enqueue* and *Dequeue* with amortized analysis.
  - *Enqueue*
    - 将入队元素压入栈A。
  - *Dequeue*
    - 若栈B为空，将栈依次将栈A中元素出栈，然后压入栈B，直至栈A为空（或剩余一个元素），将栈B顶端元素出栈（或将栈A剩余元素出栈）。
    - 若栈B不为空，直接将栈B顶端元素出栈。