1-4 基本的算法结构

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2017 年 11 月 06 日

Longest Monotone Subsequence

`while-do`

Longest Monotone Subsequence

ES 24.8: Longest Monotone Subsequence

Write a computer program that takes as its input a sequence of distinct integers and returns as its output the length of a longest monotone subsequence.

Understanding this problem:

Subsequence *vs.* substring

Monotone increasing *vs.* decreasing   strictly *vs.* non-strictly

Longest existence? uniqueness?

The Length *vs.* the subsequence itself

ES 24.8: Longest (Strictly) Increasing Subsequence (LIS)

- Given an integer array $A[0 \ldots n-1]$
- To find the length L of an LIS

$$0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15 \implies 0, 2, 6, 9, 11, 15$$

学生反馈： 这道题为什么放在 “Pigeonhole Principle” 这一章?

Theorem (Erdős-Szekeres Theorem)

Let n be a positive integer. Every sequence of n^2+1 distinct integers must contain a monotone subsequence of length $n+1$.

Q : 这道题与 (强) 数学归纳法有什么关系?

B.S. $P(0)$

I.H. $P(0) \cdots P(i-1)$

I.S. $P(0) \cdots P(i-1) \to P(i)$

$P(i)$ 是什么?

$$\boxed{P(i) : \text{the length of an LIS in } A[0 \cdots i].}$$

$$L = P(n-1)$$

$$P(0) = 1$$

$$P(0) \cdots P(i-1) \to P(i)?$$

$$P(i) = \max\{P(i-1), \max_{\substack{0 \le j < i \\ A[j] < A[i]}} \{P(j) + 1\}\}$$

$P(i)$: the length of an LIS *ending at* $A[i]$.

$$L = \max_{0 \le i < n} P(i)$$

$$P(0) = 1$$

$$P(0) \cdots P(i-1) \to P(i)?$$

$$P(i) = \max_{\substack{0 \le j < i \\ A[j] < A[i]}} \{P(j) + 1\}$$

$P(0) = 1;$

```
for (int i = 1; i < n; ++i)   // How much time?
```

$$P(i) = \max_{\substack{0 \le j < i \\ A[j] < A[i]}} \{P(j) + 1\}$$

```
return
```

$$L = \max_{0 \le i < n} P(i);$$

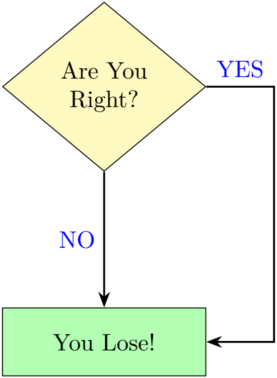
```
// How much space?
```

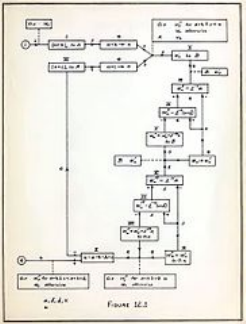
1-4 作业习题选讲

ES: Chapter 2

Flowcharts

How to Argue with Your Girlfriend?

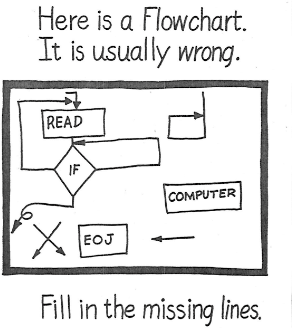


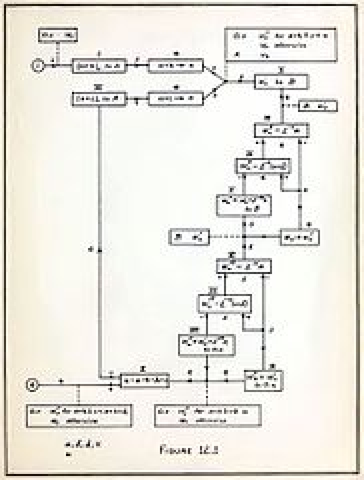


We feel certain that a moderate amount of experience with this stage of coding suffices to remove from it all difficulties, and to make it a perfectly routine operation.

— John von Neumann and Herman Goldstine, late 1940s


我的内心几
乎是崩溃的


Here is a Flowchart.
It is usually wrong.
READ
IF
COMPUTER
EOJ
Fill in the missing lines.



Flowcharts Considered Harmful.

Just my opinion...

Draw it when it does help

OR you have to.

Simulations

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(a) "for-do" by "while-do"

```
for (int i = 0; i < N; ++i) // not general!
  statement
```

```
int i = 0;
while (i < N)
  statement
  ++i
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(a) "`for-do`" by "`while-do`"

```
for (init; cond; inc)
  statement
```

```
init;
while (cond)
  statement
  inc
```

Whether to use "while" or "for" is largely a matter of personal preference.

— K&R C Bible

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
```

```
while (A)
  B
  ¬ A // Wrong: side effects?
```

```
flag = 1
while (A && flag)
  B
  flag = 0
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) "`if-then` & `if-then-else`" by "`while-do`"

```
if (A)
  B
else
  C
```

```
flag_if = 1
while (A && flag_if)
  B  // Wrong: side effects?
  flag_if = 0
flag_else = 1
while (¬ A && flag_else)
  C
  flag_else = 0
```

```
flag = 1
while (A && flag)
  B
  flag = 0
// ¬A not necessary
while (¬ A && flag)
  C
  flag = 0
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(c) "while-do" by "if-then & goto"

(d) "while-do" by "repeat-until & if-then"

```
while (A)
  B
```

```
L: if (A)
     B
     goto L
```

```
if (A)   // no ''if''?
repeat
  B
until (¬ A)
```

DH 2.8: Simulations

Simulate "`while-do`" by "`if-then-else & recursive`".

```
while (A)
  B
```

```
simulateWhile() { // define function
  if (A)
    B
    simulateWhile();

  return;
}
```

(1) `A;B`
(2) `if-then`
(3) `if-then-else`
(4) `for-do`
(5) `while-do`
(6) `repeat-until`

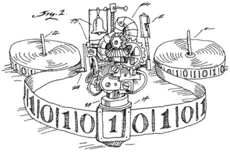
```
repeat
  B
until (¬ A)
```

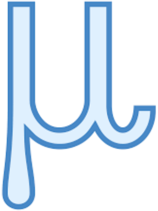
```
B
while (A)
  B
```

Theorem (“On Folk Theorems” (David Harel, 1980))

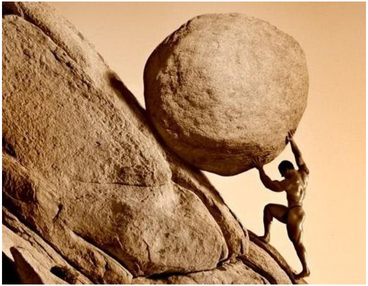
Any computable function can be computed by a “while-do” (and “;”) program (with additional Boolean variables).

Simulations for Equivalence

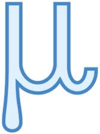




Bounded Iterations *vs.* Unbounded Iterations



Q : Why unbounded iterations?



μ-Recursive Functions

$$\color{red}{\mu} y(g(x,y)) = \Big(\operatorname*{argmin}_y g(x,y) = 0 \Big)$$

Unbounded iterations: "`while-do`"

Theorem (Ackermann Function)

The Ackermann function is μ-recursive but not primitive recursive (which contains bounded iterations.).

DH 2.4: Bounded Iteration

Given a list L of N integers, to produce in S and P the sum of the even numbers in L and the product of the odd ones, respectively.

```
int S = 0, P = 1;
for (int i = 0; i < N; ++i) {
  if (L(i) % 2 == 0)
    S += L(i);
  else
    P *= L(i);
}
```

DH 2.1: Salary Summation

$N - 1$ vs. N iterations

DH 2.7: Compute $n!$

Write algorithms that compute $n!$, given a non-negative integer n.

(a) Using iteration statements.

(b) Using recursion.

```
int P = 1;
for (int i = 2; i <= n; ++i) {
  P *= i;
}
```

```
int recursive-factorial(int n) { // define function
  if (n == 0)
    return 1;
    // NOT: return n * (n - 1)!
    else return n * recursive-factorial(n-1);
}
```


Thank
You!