

1-4 基本的算法结构

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Longest Monotone Subsequence

`while-do`

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ES 24.8: Longest Monotone Subsequence

Write a computer program that takes as its input a sequence of distinct integers and returns as its output the length of a longest monotone subsequence.

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Monotone increasing vs. decreasing

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Longest existence? uniqueness?

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Longest existence? uniqueness?

The Length vs. the subsequence itself

ES 24.8: Longest (Strictly) Increasing Subsequence (LIS)

- ▶ Given an integer array $A[0 \dots n - 1]$
- ▶ To find the length L of an LIS

0, 8, 4, 12, 2, 10, 6, 14, 1, 9, 5, 13, 3, 11, 7, 15 \implies 0, 2, 6, 9, 11, 15

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Theorem (Erdős-Szekeres Theorem)

Let n be a positive integer. Every sequence of $n^2 + 1$ distinct integers must contain a monotone subsequence of length $n + 1$.

Q: 这道题与 (强) 数学归纳法有什么关系?

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$P(i)$ 是什么?

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$$P(i) = \max\{P(i - 1), \max_{\substack{0 \leq j < i \\ A[j] < A[i]}} \{P(j) + 1\}\}$$

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$$P(i) = \max_{\substack{0 \leq j < i \\ A[j] < A[i]}} \{P(j) + 1\}$$

$P(0) = 1;$

`for (int i = 1; i < n; ++i)`

$P(i) = \max_{\substack{0 \leq j < i \\ A[j] < A[i]}} \{P(j) + 1\}$

`return L = $\max_{0 \leq i < n} P(i);$`

$P(0) = 1;$

`for (int i = 1; i < n; ++i) // How much time?`

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`return L = $\max_{0 \leq i < n} P(i);$`

$P(0) = 1;$

```
for (int i = 1; i < n; ++i) // How much time?
```

$$P(i) = \max_{\substack{0 \leq j < i \\ A[j] < A[i]}} \{P(j) + 1\}$$

```
return L = \max_{0 \leq i < n} P(i); // How much space?
```

$P(0) = 1;$

```
for (int i = 1; i < n; ++i) // How much time?
```

$$P(i) = \max_{\substack{0 \leq j < i \\ A[j] < A[i]}} \{P(j) + 1\}$$

```
return L = \max_{0 \leq i < n} P(i); // How much space?
```

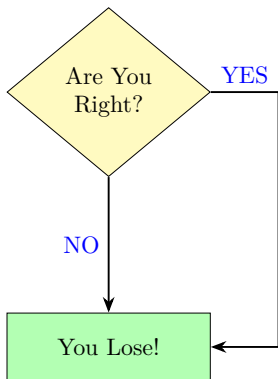


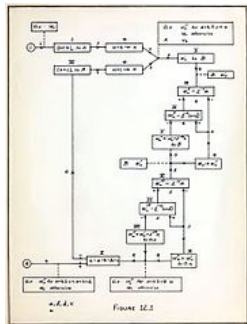
1-4 作业习题选讲

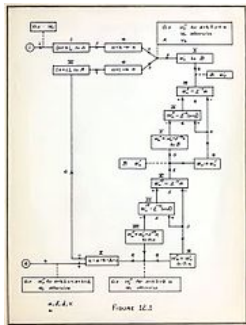
ES: Chapter 2

Flowcharts

How to Argue with Your Girlfriend?







We feel certain that a moderate amount of experience with this stage of *coding* suffices to remove from it all difficulties, and to make it a perfectly *routine operation*.

— John von Neumann and Herman Goldstine, late 1940s

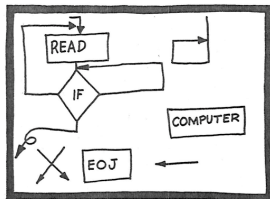


**我的内心几
乎是崩溃的**

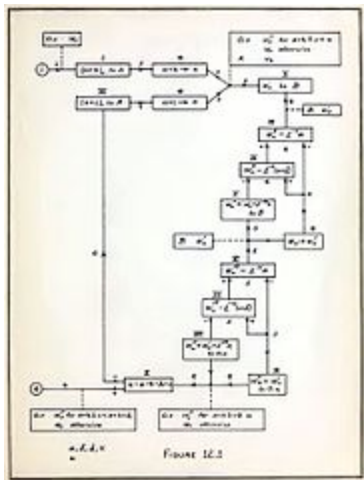


我的内心几乎是崩溃的

Here is a Flowchart.
It is usually wrong.



Fill in the missing lines.



Flowcharts Considered Harmful.

Just my opinion...

Just my opinion...

Draw it when it does help

Just my opinion...

Draw it when it does help
OR you have to.

Simulations

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(a) “for-do” by “while-do”

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```
for (int i = 0; i < N; ++i)
    statement
```

```
int i = 0;
while (i < N)
    statement
    ++i
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(a) “for-do” by “while-do”

```
for (int i = 0; i < N; ++i) // not general!  
    statement
```

```
int i = 0;  
while (i < N)  
    statement  
    ++i
```


DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(a) “for-do” by “while-do”

```
for (init; cond; inc)
    statement
```

```
init;
while (cond)
    statement
inc
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(a) “for-do” by “while-do”

```
for (init; cond; inc)
    statement
```

```
init;
while (cond)
    statement
inc
```

Whether to use “while” or “for” is largely a matter of personal preference.

— K&R C Bible

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) “if-then & if-then-else” by “while-do”

```
if (A)
  B
```

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(b) “if-then & if-then-else” by “while-do”

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if (A)
  B
```

```
while (A)
  B
  ¬ A
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) “if-then & if-then-else” by “while-do”

```
if (A)
  B
```

```
while (A)
  B
   $\neg$  A // Wrong: side effects?
```

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Perform the following simulations of some control constructs by others.

(b) “if-then & if-then-else” by “while-do”

```
if (A)
  B
```

```
while (A)
  B
   $\neg$  A // Wrong: side effects?
```

```
flag = 1
while (A && flag)
  B
  flag = 0
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) “if-then & if-then-else” by “while-do”

```
if (A)
  B
else
  C
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(b) “if-then & if-then-else” by “while-do”

```
if (A)
  B
else
  C
```

```
flag_if = 1
while (A && flag_if)
  B
  flag_if = 0
flag_else = 1
while ( $\neg$  A && flag_else)
  C
  flag_else = 0
```


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Perform the following simulations of some control constructs by others.

(b) “if-then & if-then-else” by “while-do”

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if (A)
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else
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flag_else = 1
while ( $\neg$  A && flag_else)
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  C
  flag_else = 0
```

```
flag = 1
while (A && flag)
  B
  flag = 0

while ( $\neg$  A && flag)
  C
  flag = 0
```

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Perform the following simulations of some control constructs by others.

(b) “if-then & if-then-else” by “while-do”

```
if (A)
  B
else
  C
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flag_if = 1
while (A && flag_if)
  B // Wrong: side effects?
  flag_if = 0
flag_else = 1
while ( $\neg$  A && flag_else)
  C
  flag_else = 0
```

```
flag = 1
while (A && flag)
  B
  flag = 0
//  $\neg$ A not necessary
while ( $\neg$  A && flag)
  C
  flag = 0
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(c) “while-do” by “if-then & goto”

(d) “while-do” by “repeat-until & if-then”

```
while (A)
  B
```

DH 2.5: Simulations

Perform the following simulations of some control constructs by others.

(c) “while-do” by “if-then & goto”

(d) “while-do” by “repeat-until & if-then”

```
while (A)
    B
```

```
L: if (A)
    B
    goto L
```

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Perform the following simulations of some control constructs by others.

(c) “while-do” by “if-then & goto”

(d) “while-do” by “repeat-until & if-then”

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while (A)
  B
```

```
L: if (A)
    B
    goto L
```

```
if (A)
  repeat
    B
  until ( $\neg$  A)
```

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(c) “while-do” by “if-then & goto”

(d) “while-do” by “repeat-until & if-then”

```
while (A)
  B
```

```
L: if (A)
    B
    goto L
```

```
if (A) // no 'if'?
repeat
  B
until ( $\neg$  A)
```

DH 2.8: Simulations

Simulate “while-do” by “if-then-else & recursive”.

```
while (A)  
  B
```


DH 2.8: Simulations

Simulate “while-do” by “if-then-else & recursive”.

```
while (A)
  B
```

```
simulateWhile() {
  if (A)
    B
    simulateWhile();

  return;
}
```

DH 2.8: Simulations

Simulate “while-do” by “if-then-else & recursive”.

```
while (A)
  B
```

```
simulateWhile() { // define function
  if (A)
    B
    simulateWhile();

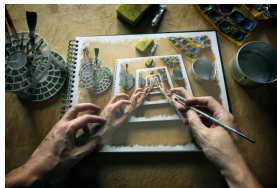
  return;
}
```

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Simulate “while-do” by “if-then-else & recursive”.

```
while (A)  
  B
```

```
simulateWhile() { // define function  
  if (A)  
    B  
    simulateWhile();  
  
  return;  
}
```



- (1) A;B
- (2) if-then
- (3) if-then-else
- (4) for-do
- (5) while-do
- (6) repeat-until

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```
repeat  
  B  
until ( $\neg$  A)
```

```
B  
while (A)  
  B
```

- (1) A;B
- (2) if-then
- (3) if-then-else
- (4) for-do
- (5) while-do
- (6) repeat-until

```
repeat
  B
until ( $\neg$  A)
```

```
B
while (A)
```

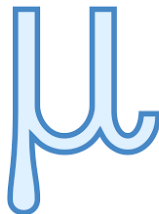
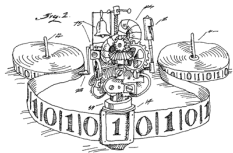
Theorem (“On Folk Theorems” (David Harel, 1980))

Any *computable function* can be computed by a “while-do” (and “;”) program (with additional Boolean variables).

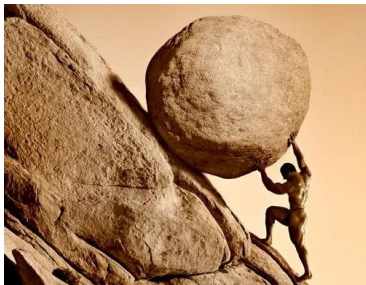




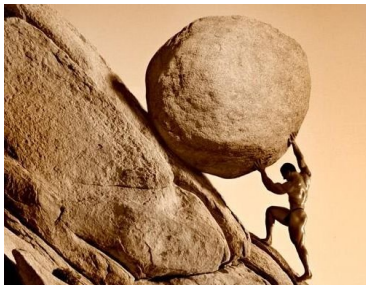
Simulations for Equivalence



Bounded Iterations vs. Unbounded Iterations



Bounded Iterations vs. Unbounded Iterations



Q : Why unbounded iterations?



μ -Recursive Functions

$$\mu y(g(x, y)) = \left(\underset{y}{\operatorname{argmin}} g(x, y) = 0 \right)$$



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Unbounded iterations: “while-do”

Theorem (Ackermann Function)

The Ackermann function is μ -recursive but not *primitive* recursive (which contains *bounded* iterations).

DH 2.4: Bounded Iteration

Given a list L of N integers, to produce in S and P the sum of the even numbers in L and the product of the odd ones, respectively.

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Given a list L of N integers, to produce in S and P the sum of the even numbers in L and the product of the odd ones, respectively.

```
int S = 0, P = 1;
for (int i = 0; i < N; ++i) {
    if (L(i) % 2 == 0)
        S += L(i);
    else
        P *= L(i);
}
```

DH 2.4: Bounded Iteration

Given a list L of N integers, to produce in S and P the sum of the even numbers in L and the product of the odd ones, respectively.

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```

DH 2.1: Salary Summation

$N - 1$ vs. N iterations

DH 2.4: Bounded Iteration

Given a list L of N integers, to produce in S and P the sum of the even numbers in L and the product of the odd ones, respectively.

```
int S = 0, P = 1;
for (int i = 0; i < N; ++i) {
    if (L(i) % 2 == 0)
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    else
        P *= L(i);
}
```

DH 2.1: Salary Summation

$N - 1$ vs. N iterations



DH 2.7: Compute $n!$

Write algorithms that compute $n!$, given a non-negative integer n .

- (a) Using iteration statements.
- (b) Using recursion.

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Write algorithms that compute $n!$, given a non-negative integer n .

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```
int P = 1;
for (int i = 2; i <= n; ++i) {
    P *= i;
}
```

DH 2.7: Compute $n!$

Write algorithms that compute $n!$, given a non-negative integer n .

- (a) Using iteration statements.
- (b) Using recursion.

```
int P = 1;
for (int i = 2; i <= n; ++i) {
    P *= i;
}
```

```
int recursive-factorial(int n) {
    if (n == 0)
        return 1;

    else return n * recursive-factorial(n-1);
}
```

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Write algorithms that compute $n!$, given a non-negative integer n .

- (a) Using iteration statements.
- (b) Using recursion.

```
int P = 1;
for (int i = 2; i <= n; ++i) {
    P *= i;
}
```

```
int recursive-factorial(int n) { // define function
    if (n == 0)
        return 1;

    else return n * recursive-factorial(n-1);
}
```

DH 2.7: Compute $n!$

Write algorithms that compute $n!$, given a non-negative integer n .

- (a) Using iteration statements.
- (b) Using recursion.

```
int P = 1;
for (int i = 2; i <= n; ++i) {
    P *= i;
}
```

```
int recursive-factorial(int n) { // define function
    if (n == 0)
        return 1;
    // NOT: return n * (n - 1)!
    else return n * recursive-factorial(n-1);
}
```


Thank
You!