问题与讨论
2014/3/27

- 图构造问题
- 给定n个自然数: d1, d2, … , dn, 尝试设计一个算法判断是否存在一个无向图（图中的边没有方向性），该图中顶点的度数的值正好为d1, d2, … , dn. 无向图中顶点的度数就是与该顶点相连的边数。要求图中任意两个顶点之间最多只能有一条边；也没有连接同一顶点的边。

- 给出算法的思路和原理；
- 尝试证明算法的部分正确性和完全正确性；
- 分析算法的时间复杂度；
- 尝试给**图构造问题**一个下界，并试着**完整地**证明这个下界。

The Havel-Hakimi Algorithm

Take as input a degree sequence S and determine if that sequence is graphical

That is, can we produce a graph with that degree sequence?

Assume the degree sequence is S $S = d_1, d_2, d_{3,} \cdots, d_n$

$d_i \geq d_{i+1}$

1. If any $d_i \geq n$ then fail
2. If there is an odd number of odd degrees then fail
3. If there is a $d_i < 0$ then fail
4. If all $d_i = 0$ then report success
5. Reorder S into non - increasing order
6. Let $k = d_1$
7. Remove d_1 from S.
8. Subtract 1 from the first k terms remaining of the new sequence
9. Go to step 3 above

Note: steps 1 and 2 are a pre-process

3. If there is a $d_i < 0$ then fail
4. If all $d_i = 0$ then report success
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S = 4,3,3,3,1

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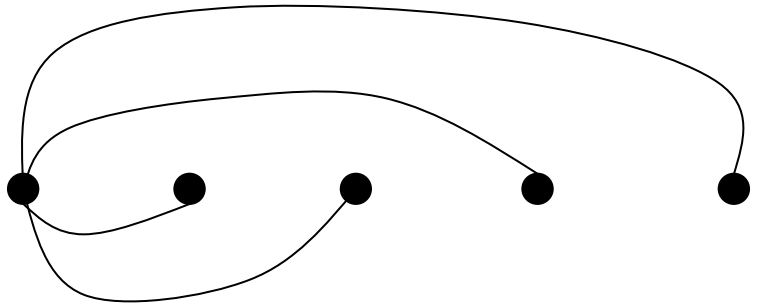
S = 4,3,3,3,1

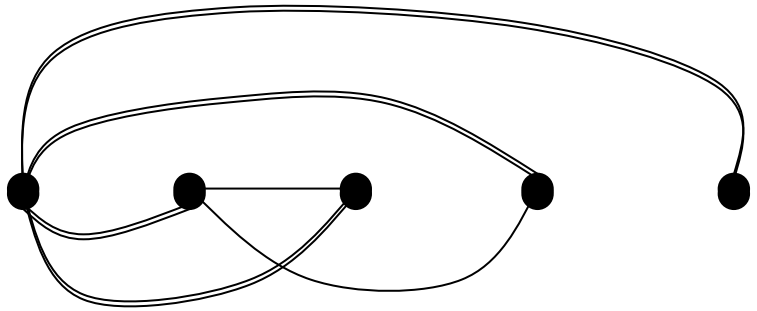


S = 2,2,2,0

3. If there is a $d_i < 0$ then fail
4. If all $d_i = 0$ then report success
5. Reorder S into non-increasing order
6. Let $k = d_1$
7. Remove d_1 from S.
8. Subtract 1 from the first k terms remaining of the new sequence
9. Go to step 3 above

S = 4,3,3,3,1

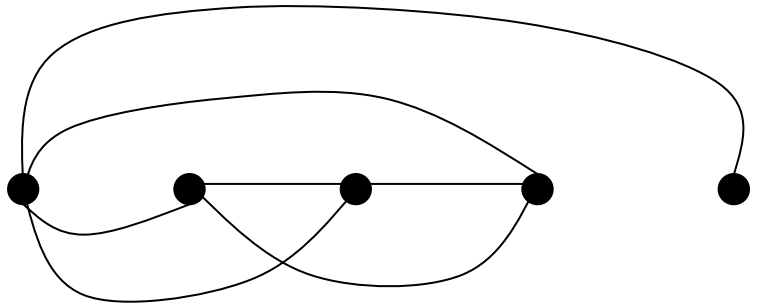
S = 1,1,0



3. If there is a $d_i < 0$ then fail
4. If all $d_i = 0$ then report success
5. Reorder S into non-increasing order
6. Let $k = d_1$
7. Remove d_1 from S.
8. Subtract 1 from the first k terms remaining of the new sequence
9. Go to step 3 above

S = 4,3,3,3,1

S = 0,0



3. If there is a $d_i < 0$ then fail
4. If all $d_i = 0$ then report success
5. Reorder S into non-increasing order
6. Let $k = d_1$
7. Remove d_1 from S.
8. Subtract 1 from the first k terms remaining of the new sequence
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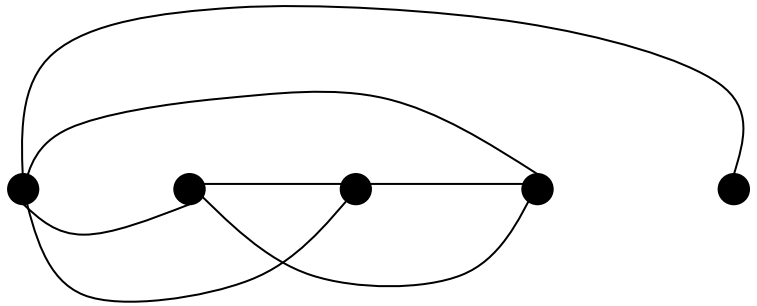
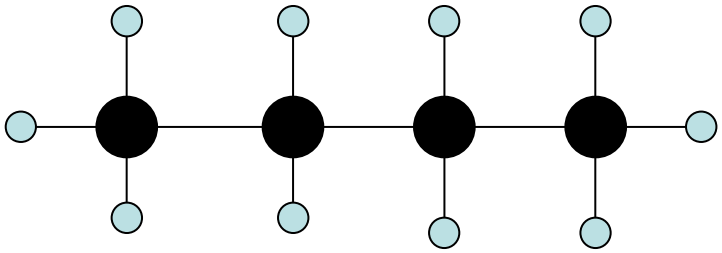
S = 4,3,3,3,1

Report Success



4,4,4,4,1,1,1,1,1,1,1,1,1,1

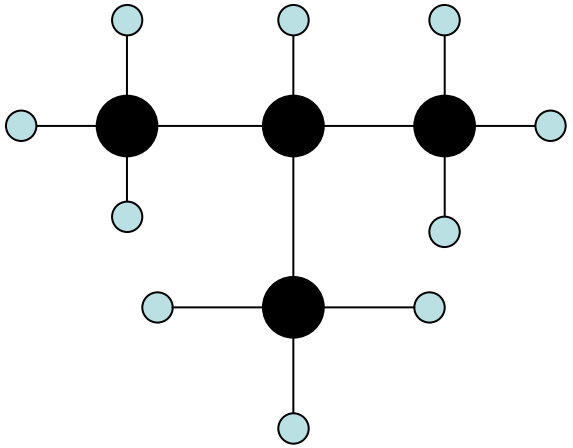
C_4H_{10}



Alternatively

4,4,4,4,1,1,1,1,1,1,1,1,1,1

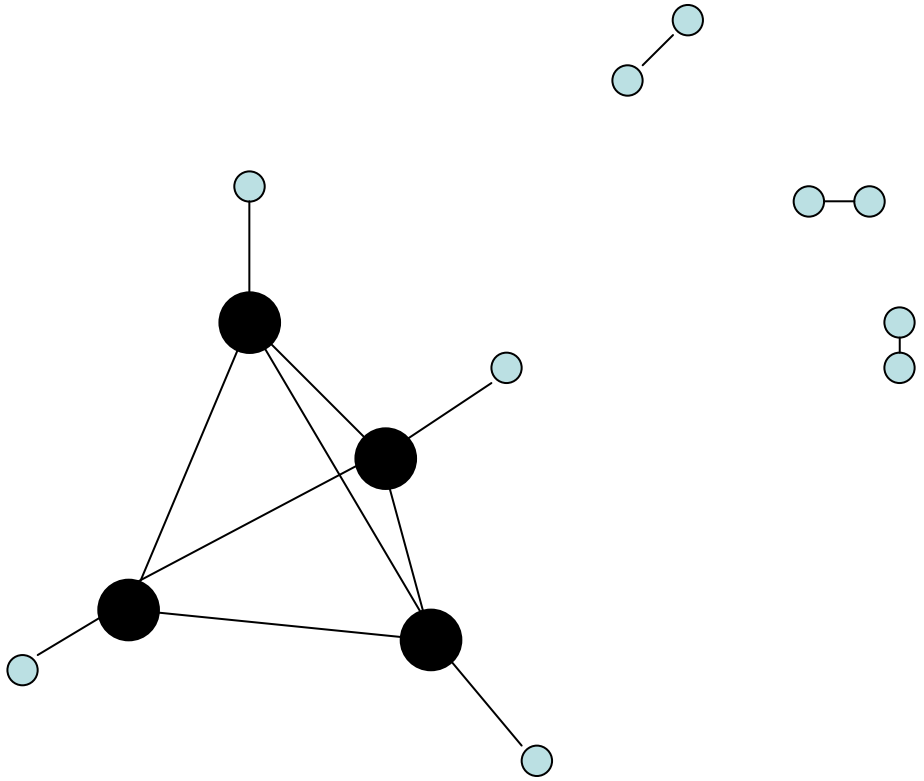
C_4H_{10}



Havel-Hakimi produces the following

4,4,4,4,1,1,1,1,1,1,1,1,1,1

$$C_4H_4 + 3H_2$$



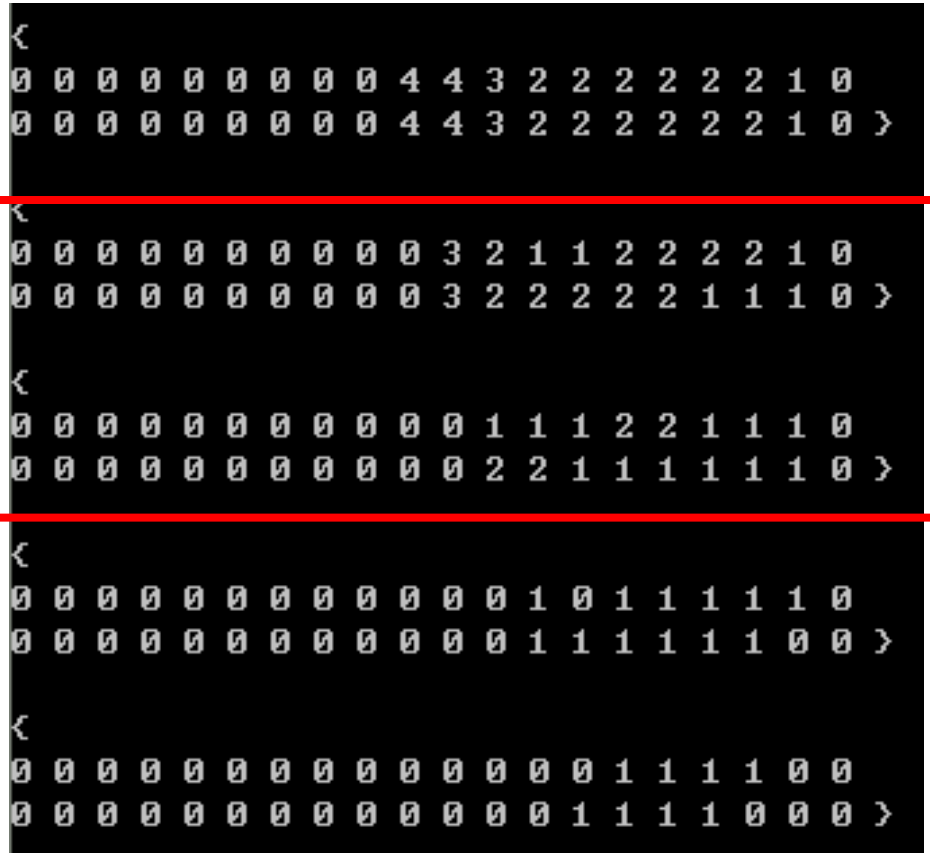
The hypothetical hydrocarbon **Vinylacetylene**

So?
(the question from hell)

Well, we have demonstrated that the HH algorithm doesn't always produce
A connected graph.

We have also shown that by representing molecules as simple graphs and using an algorithm to model this graph we might get some unexpected results, maybe something new!

- Graph Building Problem:
- Given a list of n natural numbers d1, d2, … , dn, show how to decide in polynomial time whether there exists an undirected graph whose degree are precisely the number d1, d2, … , dn. (G should not contain multiple edges between the same pair of vertices or “loop” edges with both endpoints equal to the same node).
- 测试数据： (格式：20个度，答案)
- 20 11 18 3 19 14 7 10 7 19 14 19 5 13 14 13 15 12 8 14 9 Yes
- 20 14 5 12 4 19 2 14 5 17 8 10 8 18 15 15 11 16 19 12 12 Yes
- 20 5 16 13 10 15 19 6 17 5 4 19 4 6 12 1 12 14 4 12 18 No
- 20 2 13 10 14 15 3 8 18 2 12 9 7 2 12 11 4 19 7 13 11 Yes
- 20 13 14 1 8 2 10 18 18 13 6 16 12 9 10 4 5 1 2 6 14 No
- 20 13 15 12 17 6 1 4 16 7 5 1 7 11 4 7 4 16 5 9 16 No
- 8 13 16 9 8 2 11 7 10 18 3 6 14 14 5 7 16 18 11 8 Yes
- 20 6 17 7 18 9 18 9 11 15 11 11 13 12 18 18 10 11 3 3 2 No
- 20 14 6 6 16 2 19 7 12 10 6 7 10 17 14 1 17 11 15 13 17 Yes
- 20 3 7 1 6 14 1 10 3 15 14 12 9 2 9 19 15 16 14 14 6 No
- 20 14 13 5 16 11 7 13 16 19 8 5 19 1 9 6 14 4 18 2 16 No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0 0 0 0 0 0 0 0 0 4 4 3 2 2 2 2 2 2 1 0 }
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