

- 书面讲解
 - TC第4.1节练习5
 - TC第4.3节练习3、7
 - TC第4.4节练习2、8
 - TC第4章问题1、2、4

TC第4.1节练习5

```
int sequence(std::vector<int>& numbers)
{
    // Initialize variables here
    int max_so_far = numbers[0], max_ending_here = numbers[0];
    size_t begin = 0;
    size_t begin_temp = 0;
    size_t end = 0;
    // Find sequence by looping through
    for(size_t i = 1; i < numbers.size(); i++)
    {
        // calculate max_ending_here
        if(max_ending_here < 0)
        {
            max_ending_here = numbers[i];
            begin_temp = i;
        }
        else
        {
            max_ending_here += numbers[i];
        }
        // calculate max_so_far
        if(max_ending_here > max_so_far )
        {
            max_so_far = max_ending_here;
            begin = begin_temp;
            end = i;
        }
    }
    return max_so_far ;
}
```

动态规划

- max_so_far: A[1..j]上的最大值
- max_ending_here: 最大的A[i...j+1]
- 两者中较大者为A[1..j+1]上的最大值

注意点:

- 1、边界情况的处理: 所有A[i]都是负数怎么办?
- 2、不论如何, 算法最后一定要返回一个结果。

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
13	-3	-25	20	-3	-16	-23	18	20	-7	12	-5	-22	15	-4	7

抛弃和为负数的前缀

TC第4.3节练习3

- 大胆缩放、正确缩放

$$T(n) \geq 2 \left(c \left\lfloor \frac{n}{2} \right\rfloor \lg \left\lfloor \frac{n}{2} \right\rfloor \right) + n \geq 2c \frac{n-1}{2} \lg \frac{n}{4} + n$$

$$= cn \lg n - c \lg n - 2cn + 2c + n = cn \lg n - \frac{1}{4} \lg n + \frac{1}{2} n + \frac{1}{2} \geq cn \lg n$$

(c=1/4)

- 不要忘记boundary condition

TC第4.3节练习7

如果欲证 $T(n) \leq cn^{\log_3 4}$

$$T(n) \leq \dots \leq cn^{\log_3 4} + n \geq cn^{\log_3 4}, \quad \text{fails}$$

改为欲证 $T(n) \leq cn^{\log_3 4} - \frac{1}{4}n$

$$T(n) \leq \dots \leq cn^{\log_3 4} + n - n = cn^{\log_3 4}, \quad \text{这样对吗?}$$

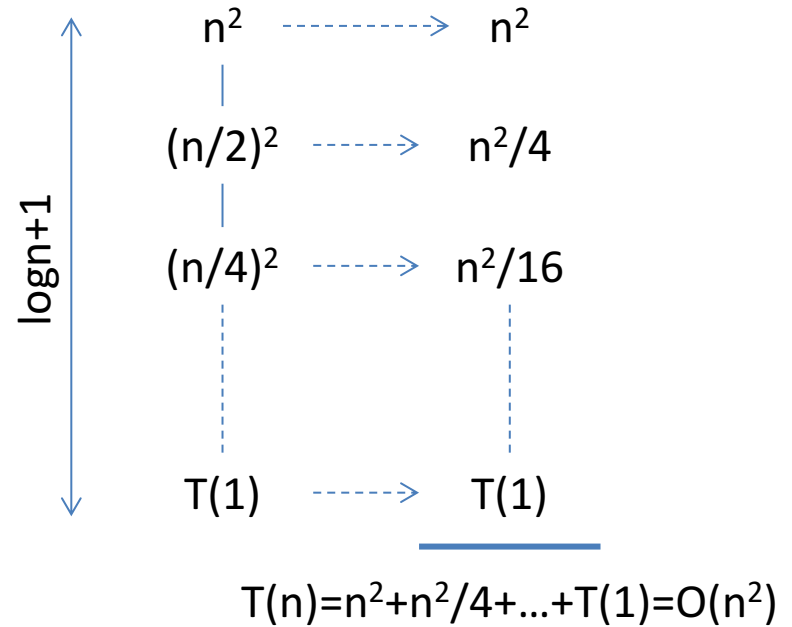
改为欲证 $T(n) \leq cn^{\log_3 4} + dn$

$$T(n) \leq \dots \leq cn^{\log_3 4} + \left(\frac{4}{3}d + 1\right)n \leq cn^{\log_3 4} + dn, \quad \text{只要 } \frac{4}{3}d + 1 \leq d$$

TC第4.4节练习2

- recursion tree的组成元素缺一不可

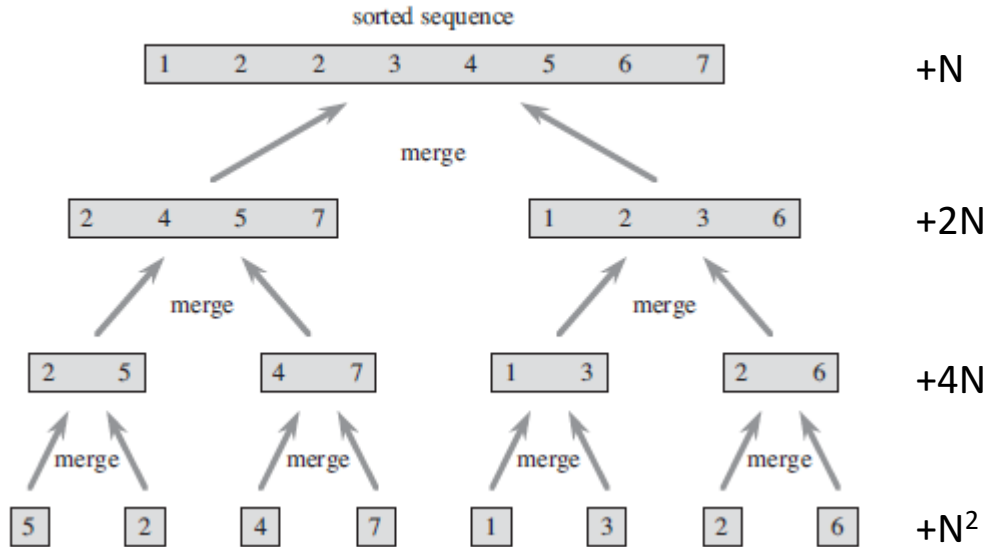
- 如何证明 $T(n) \in O(f(n))$?
 - 对于 $n > n_0$, $T(n) \leq c_1 f(n)$
- 如何证明 $T(n) \in \Theta(f(n))$?
 - 对于 $n > n_0$, $c_1 f(n) \leq T(n) \leq c_2 f(n)$



- 什么是substitution method?
 - 数学归纳法

TC第4章问题2b

- $T(n) = 2T(n/2) + \Theta(n) + p(n)$
Merge Pass
- By pointer/By copying subrange
 - $T(n) = 2T(n/2) + \Theta(n) = \Theta(n \lg n)$
- By copying: $p(n) = N$

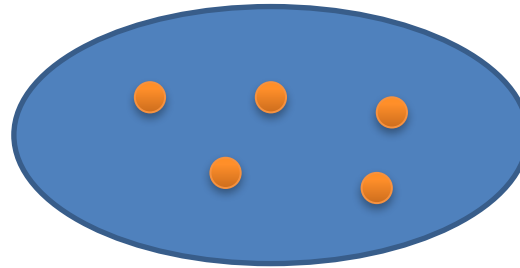


$$T(n) = \Theta(n \lg n + n^2) = \Theta(n^2)$$

- 教材讨论
 - CS第5章第1、2、3、4节

问题1: probability

- 你理解这些概念了吗?
 - Sample space
 - Element
 - Event
 - Probability weight
 - Probability



- 你能基于这些概念解释probability distribution function的三个条件吗?
 1. $P(A) \geq 0$ for any $A \subseteq S$.
 2. $P(S) = 1$.
 3. $P(A \cup B) = P(A) + P(B)$ for any two disjoint events A and B .

问题1: probability (续)

- 在这些例子中，sample space、element、event分别是什么？
 - The probability of getting at least 1 head in 5 flips of a coin.
 - The probability of getting a total of 6 or 7 on the 2 dice.
 - The probability that all 3 keys hash to different locations (among 20).
- 你能给出它们的答案吗？（真能吗？）

问题1: probability (续)

- 你理解uniform probability distribution了吗?

Theorem 5.2 Suppose P is the uniform probability measure defined on a sample space S . Then for any event E ,

$$P(E) = |E|/|S|,$$

the size of E divided by the size of S .

- 现在你能给出之前几题的答案了吗?
 - The probability of getting at least 1 head in 5 flips of a coin.
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- uniform probability distribution为计算带来了怎样的便利?

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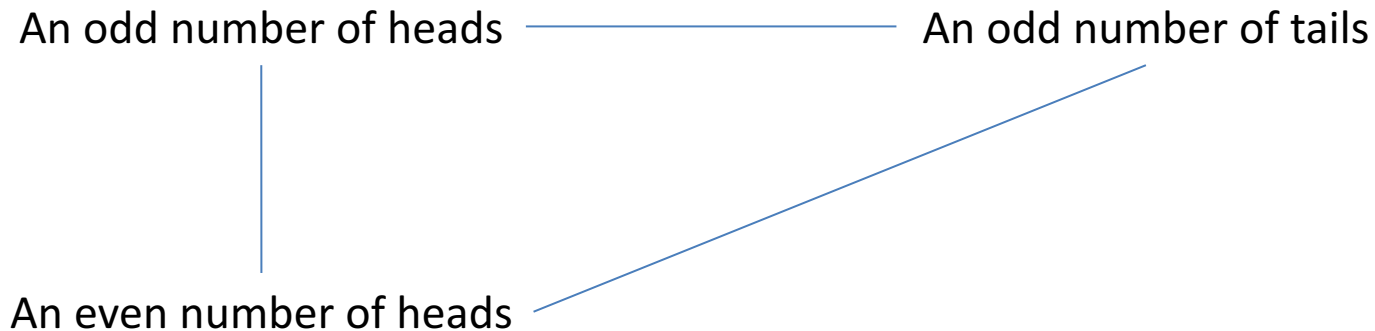
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- uniform probability distribution为计算带来了怎样的便利?
 - probability \rightarrow counting

问题1: probability (续)

- What is the probability of an odd number of heads in three tosses of a coin? (假设是uniform probability distribution)
 - 如何利用这个三角形快速求解?



- 如果不是uniform probability distribution, 怎么办?

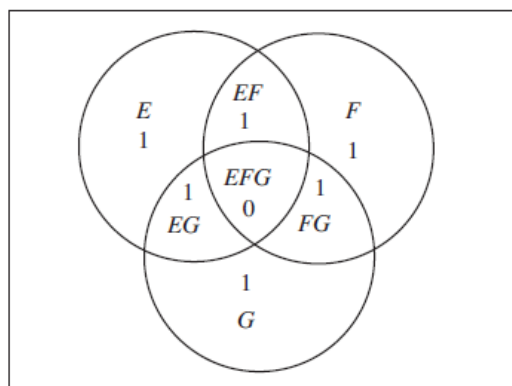
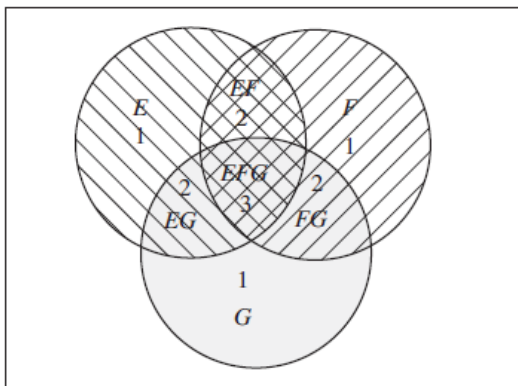
问题1: probability (续)

- Which is more likely, or are both equally likely?
 - Drawing an ace and a king when you draw two cards from among the 13 spades, or drawing an ace and a king when you draw two cards from an ordinary deck of 52 playing cards?
 - Drawing an ace and a king of the same suit when you draw two cards from a deck, or drawing an ace and a king when you draw two cards from among the 13 spaces?

问题2: the principle of inclusion and exclusion

- 你理解这两个图的含义了吗?

$$- P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(F \cap G) + P(E \cap F \cap G)$$



- 你读懂这个公式了吗?

$$P\left(\bigcup_{i=1}^n E_i\right) = \sum_{k=1}^n (-1)^{k+1} \sum_{\substack{i_1, i_2, \dots, i_k: \\ 1 \leq i_1 < i_2 < \dots < i_k \leq n}} P(E_{i_1} \cap E_{i_2} \cap \dots \cap E_{i_k})$$

问题2: the principle of inclusion and exclusion (续)

- How many functions from an m -element set M to an n -element set N map nothing to at least one element of N ?
 - Sample space?
 - Element?
 - Event?

$$\left| \bigcup_{i=1}^n E_i \right| = \sum_{k=1}^n (-1)^{k+1} \sum_{\substack{i_1, i_2, \dots, i_k: \\ 1 \leq i_1 < i_2 < \dots < i_k \leq n}} |E_{i_1} \cap E_{i_2} \cap \dots \cap E_{i_k}|$$

E_i 是什么? 在这里如何计算?

问题2: the principle of inclusion and exclusion (续)

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不映射到*i*的函数集合

$\binom{m}{k} (m-k)^n$

问题2: the principle of inclusion and exclusion (续)

- In how many ways may you distribute k identical apples to n children so that no child gets more than m ?

问题2: the principle of inclusion and exclusion (续)

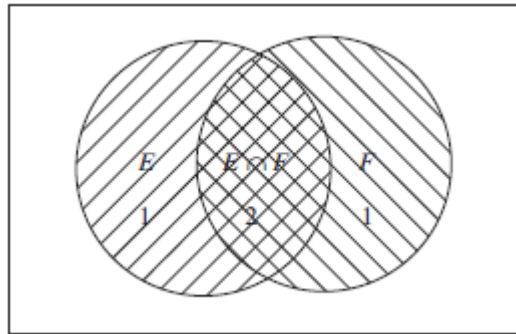
- In how many ways may you distribute k identical apples to n children so that no child gets more than m ?

$$\binom{k + (n - 1)}{n - 1} - \sum_{i=1}^n (-1)^{i+1} \binom{n}{i} \binom{k - (m + 1)i + (n - 1)}{n - 1}$$

问题3: conditional probability

- 你能结合Venn图解释条件概率的定义吗?

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$



- 你能结合图解释独立性吗?

$$P(E|F) = P(E)$$

- 你能自己推导出这两个定理吗?

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Theorem 5.4 Suppose E and F are events in a sample space. Then E is independent of F if and only if $P(E \cap F) = P(E)P(F)$.

问题3: conditional probability (续)

$$P(x_i = a_i | x_1 = a_1, \dots, x_{i-1} = a_{i-1}) = P(x_i = a_i)$$

- 你理解independent trials process了吗?

Exercise 5.3-7 Suppose we draw a card from a standard deck of 52 cards, discard it (i.e. we do not replace it), draw another card and continue for a total of ten draws. Is this an independent trials process?

- 为什么这不是一个independent trials process?
- 为这个过程绘制tree diagram, 并计算: 第*i*张抽到梅花A的概率是多少?
- 如果是independent trials process, 其tree diagram有什么特征?

问题3: conditional probability (续)

- A nickel, two dimes, and two quarters are in a cup. We draw three coins, one at a time, without replacement.
 - Draw the probability tree which represents the process.
 - Use the tree to determine the probability of getting a nickel on the last draw.
 - Use the tree to determine the probability that the first coin is a quarter, given that the last coin is a quarter.

问题4: random variables

- 你理解这些概念了吗? 能自己举个例子吗?
 - Random variable
 - Expected value
 - $E(X + Y) = E(X) + E(Y)$
- 你能直观解释这两种定义为什么等价吗?

$$E(X) = \sum_{i=1}^k x_i P(X = x_i)$$

$$E(X) = \sum_{s:s \in S} X(s)P(s)$$

问题4: random variables (续)

- 根据期望的定义求解（只需列出算式）
 - How many sixes do we expect to see on top if we roll 24 dice?
 - What is the expected number of times we need to roll two dice until we get a 7?

问题4: random variables (续)

- A student is taking a true-false test and guessing when he doesn't know the answer. We are going to compute a score by subtracting a percentage of the number of incorrect answers from the number of correct answers. That is, for some number y , the student's corrected score will be

$(\text{number of corrected answers}) - y(\text{number of incorrect answers})$

When we convert this “corrected score” to a percentage score, we want its expected value to be the percentage of the material being tested that the student knows. How can we do this?