4-8 Formalization

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JH 2.3.1.8

Design a representation of weighted graphs, where weights are some positive integers, using the alphabet $\{0, 1, \#\}$.

Represent Weighted Graph with $\{0, 1, \#\}$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

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$\bar{a}_{ij} \in \{0, 1\}^+$ is the bianary representation of a_{ij}

JH 2.3.3.8

Describe a polynomial-time verifier for

1. HC
2. VC
3. CLIQUE

Hamilton Cycle Problem

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A verifier checks the followings:

- $(c_i, c_{i+1}) \in w.E$, for $1 \leq i < n$
- $(c_n, c_1) \in w.E$
- $c_i \neq c_j$ for $1 \leq i, j \leq n, i \neq j$

Vertex Cover Problem

$$VCP = \{u\#w \in \{0,1,\#\}^+ \quad | u \in \{0,1\}^+ \text{ and } w \text{ represents a graph that contains a vertext cover of size Number(u)}\}$$

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Given a graph w, and a certificate $c \subseteq w.V$
A verifier checks the following:

- $|c| = Number(u)$
- c covers all vertexes of w, i.e. $c \cup N(c) = w.V$

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A verifier checks the following:

- $|c| = Number(x)$
- Every pair $v_i, v_j \in c$ is connected.


Thank
You!