反馈与讨论

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Important Concepts, Formulas, and Theorems

1. *Sample space.* A *sample space* is the set of possible outcomes of a process.
2. *Event.* A set of elements in a sample space is called an *event*.
3. *Disjoint.* Two events E and F are said to be *disjoint* if $E \cap F = \emptyset$.
4. *Probability.* To compute probabilities, we assign a weight to each element of the sample space so that the weight represents what we believe to be the relative likelihood of that outcome. We must follow two rules in assigning weights. First, the weights must be nonnegative numbers, and second, the sum of the weights of all the elements in a sample space must be 1. We define the *probability* $P(E)$ of the event E to be the sum of the weights of the elements of E. The function P is called a *probability measure.*
5. *The axioms of probability.* A probability measure on a finite sample space must satisfy the following three rules. (Alternately, these rules could be used to define what we mean by probability.)
   a. $P(A) \geq 0$ for any $A \subseteq S$.
   b. $P(S) = 1$.
   c. $P(A \cup B) = P(A) + P(B)$ for any two disjoint events A and B.

6. *Probability distribution.* A function that assigns a probability to each member of a sample space is called a (discrete) *probability distribution.*
7. *Complement.* The *complement* of an event E in a sample space S, denoted by $S - E$, is the set of all outcomes in S but not in E. We say that the events E and F are *complementary* events if E is the complement of F in S.
8. *The probabilities of complementary events.* If two events E and F are complementary, then

$$P(E) = 1 - P(F).$$

10. *Uniform probability distribution.* We say P is the *uniform probability measure* or *uniform probability distribution* when we assign the same probability to all members of our sample space.
11. *Computing probabilities with the uniform distribution.* Suppose P is the uniform probability measure defined on a sample space S. Then for any event E, we have $P(E) = |E|/|S|$, which is the size of E divided by the size of S. This *does not* apply to general probability distributions.

6. Two pennies, a nickel, and a dime are placed in a cup. You draw a first coin and a second coin.

   a. Assuming you are sampling without replacement (that is, you don't replace the first coin before taking the second), write the sample space of all ordered pairs of letters P, N, and D that represent the outcomes. What would you say are the appropriate weights for the elements of the sample space?

   b. What is the probability of getting 11 cents?

10. How many five-card hands chosen from a standard deck of playing cards consist of five cards in a row (such as the nine of diamonds, ten of clubs, jack of clubs, queen of hearts, and king of spades)? Such a hand is called a straight. What is the probability that a five-card hand is a straight? Explore whether you get the same answer by using five-element sets as your model of hands or five-element permutations as your model of hands.

11. A student taking a 10-question, true-false diagnostic test knows none of the answers and must guess at each one. Compute the probability that the student gets a score of 80 or higher. What is the probability that the grade is 70 or lower?

12. A die is made of a cube with a square painted on one side, a circle on two sides, and a triangle on three sides. If the die is rolled twice, what is the probability that the two shapes you see on top are the same?

13. Are the following two events equally likely? Event 1 consists of drawing an ace and a king when you draw two cards from among the 13 spades in a deck of cards. Event 2 consists of drawing an ace and a king when you draw two cards from the whole deck.