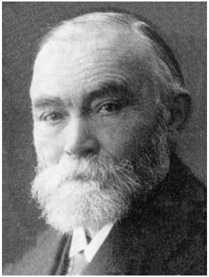
1-8 集合及其运算

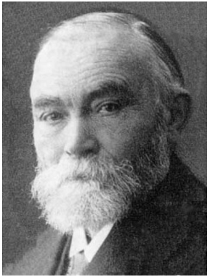
魏恒峰

hfwei@nju.edu.cn

2017 年 12 月 04 日



Gottlob Frege (1848–1925)



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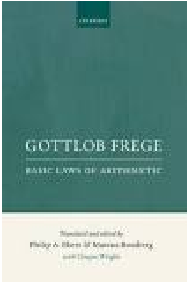
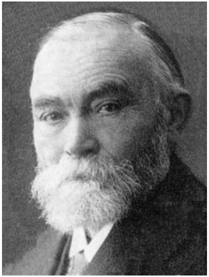


“Basic Laws of Arithmetic”



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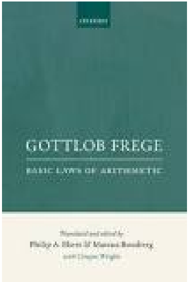
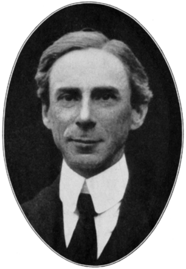
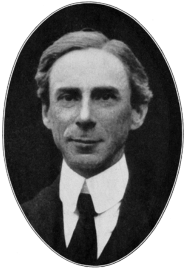


“Basic Laws of Arithmetic”

对于一个科学工作者来说，最不幸的事情莫过于：当他的工作接近完成时，却发现那大厦的基础已经动摇。 ——《附录二》



Bertrand Russell (1872–1970)



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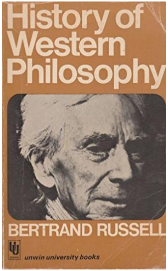

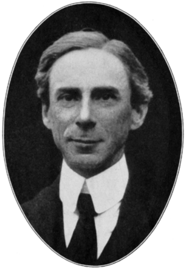

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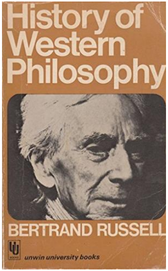

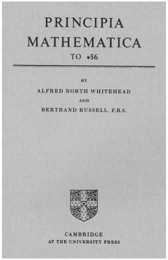

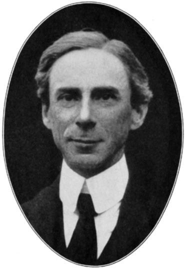

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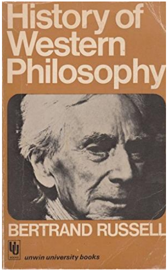

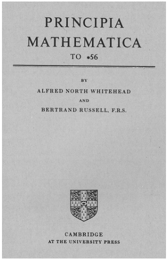

我们将集合理解为任何将我们思想中那些确定而彼此独立的对象放在一起而形成的聚合。

– *Cantor*《超穷数理论基础》

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Theorem (概括原则)

$$\forall \psi(x)\ \exists X : X = \{x \mid \psi(x)\}.$$

Definition (Russell's Paradox)

$$\psi(x) = \text{“}x \notin x\text{”}$$

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$$R = \{x \mid x \notin x\}$$

$$Q : R \in R\,?$$

Q： 既然朴素集合论存在悖论，你是如何做作业的?


YOU HAVE
TO FIX IT!

Solution: $\{x \mid x \notin x\}$ **does not exist!**

tan90°

A Little Axiomatic Set Theory (ZFC)

Ernst Zermelo (1871–1953)

Abraham Fraenkel (1891–1965)

Definition (Axiom Schema of Separation)

$$\forall \psi(x) : \Big(\forall X \exists Y : Y = \{x \in X \mid \psi(x)\} \Big).$$

$$\psi(x) = x \notin x$$

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Theorem

$$R = \{x \mid x \notin x\}$$ **is not a set.**

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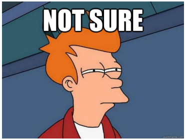
Proof.

For Your Research.



□

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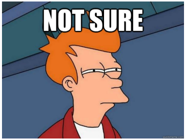

□

$$R \in R\,?$$

$$\forall X : R_X = \{x \in X \mid x \notin x\}$$

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Proof.

By contradiction.

$$\forall X : R_X = \{x \in X \mid x \notin x\}$$

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$$\{x \in C \mid x \notin x\}$$

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$$\{x \in C \mid x \notin x\} = \{x \mid x \notin x\}$$

□

$$\forall X : R_X = \{x \in X \mid x \notin x\}$$

$$Q : R_X \in R_X?$$

Theorem

There is no universe set. (It is too "big" to be a set!)

$$\forall C \exists x : x \notin C.$$

Proof.

By contradiction.

$$\{x \in C \mid x \notin x\} = \{x \mid x \notin x\}$$

□

Definition ("∩")

$$\begin{aligned} A \cap B &= \{x \in A \mid x \in B\} \\ &= \{x \mid x \in A \land x \in B\} \end{aligned}$$

Definition ("\")

$$\begin{aligned} A \setminus B &= \{x \in A \mid x \notin B\} \\ &= \{x \mid x \in A \land x \notin B\} \end{aligned}$$

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We can never look for objects “not in B” unless we know where to start looking. So we use A to tell us where to look for elements not in B. – *UD (Chapter 6)*

Definition (Axiom of Extensionality)

$$\forall A \forall B \forall x (x \in A \iff x \in B) \iff A = B.$$

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$$\{a, a\} = \{a\}$$

Set Operations

$\cap \qquad \cup \qquad \setminus$

UD 7.1 (b): Distributive Property

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

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Theorem (Distributive Property (Theorem 7.1))

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Proof.

If $x \in A \cup (B \cap C)$, then $x \in A$ or $x \in B \cap C$. Suppose first that $x \in A$. Then $x \in A \cup B$ and $x \in A \cup C$. In this first case, we see that $x \in (A \cup B) \cap (A \cup C)$. Now suppose that $x \in B \cap C$. Then $x \in B$ and $x \in C$. Since $x \in B$, we see that $x \in A \cup B$. Since we also have $x \in C$, we see that $x \in A \cup C$. Therefore, $x \in (A \cup B) \cap (A \cup C)$ in this case as well. In either case $x \in (A \cup B) \cap (A \cup C)$ and we may conclude that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

To complete the proof, we must now show that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$. So if $x \in (A \cup B) \cap (A \cup C)$, then $x \in A \cup B$ and $x \in A \cup C$. It is, once again, helpful to break this into two cases, since we know that either $x \in A$ or $x \notin A$. Now if $x \in A$, then $x \in A \cup (B \cap C)$. If $x \notin A$, then the fact that $x \in A \cup B$ implies that x must be in B. Similarly, the fact that $x \in A \cup C$ implies that x must be in C. Therefore, $x \in B \cap C$. Hence $x \in A \cup (B \cap C)$. In either case $x \in A \cup (B \cap C)$ and we may conclude that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$.

Since we proved containment in both directions we may conclude that the two sets are equal. ■

UD 7.1 (c): DeMorgan's Law

Let X denote a set, and $A, B \subseteq X$.

$$X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$$

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UD $7.1(d)$

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For any given $x, \cdots$

$Q: A, B \subseteq X?$ $\quad$ ("$\Leftarrow: X = \emptyset$")

Equivalence: UD 7.8

Consider the following sets:

(i) $(A \cap B) \setminus (A \cap B \cap C)$

(ii) $A \cap B \setminus (A \cap B \cap C)$

(iii) $A \cap B \cap C^c$

(iv) $(A \cap B) \setminus C$

(v) $(A \setminus C) \cap (B \setminus C)$

(a) Which of the sets above are written ambiguously, if any?

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(none): from the left to the right

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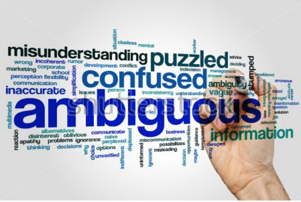

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(c) Prove that $(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$.

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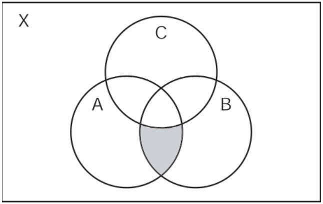

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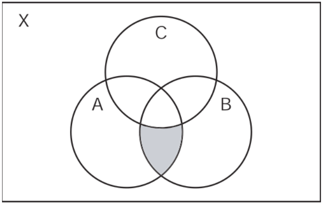

(c) Prove that $(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$.

$$A \setminus C = \{x \mid x \in A \wedge x \notin C\}$$

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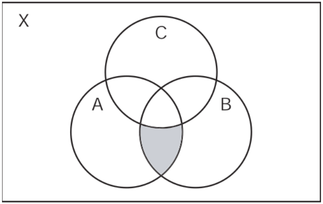

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$$A \setminus C = \{x \mid x \in A \wedge x \notin C\}$$

$$A \setminus C = A \cap C^c$$

UD 7.9

Prove that the union of two sets can be rewritten as the union of two disjoint sets.

(a) Prove that $(A \setminus B) \cap B = \emptyset$

(b) Prove that $A \cup B = (A \setminus B) \cup B$

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"太容易了，一时没反应过来"

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By contradiction.

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By contradiction.

$(A \setminus B) \cup B = \cdots$

“太容易了，一时没反应过来”

Set Family $\{A_\alpha : \alpha \in I\}$

$\bigcap \quad \bigcup$

$$\bigcup_{j=1}^{n} A_j = A_1 \cup A_2 \cup \cdots \cup A_n \qquad \bigcap_{j=1}^{n} A_j = A_1 \cap A_2 \cap \cdots \cap A_n$$

$$\bigcup_{j=1}^{n} A_j = A_1 \cup A_2 \cup \cdots \cup A_n \qquad \bigcap_{j=1}^{n} A_j = A_1 \cap A_2 \cap \cdots \cap A_n$$

$$\bigcup_{j=1}^{\infty} A_j = A_1 \cup A_2 \cup \cdots \qquad \bigcap_{j=1}^{\infty} A_j = A_1 \cap A_2 \cap \cdots$$

$$\bigcup_{\alpha \in I} A_\alpha = \{x \mid \exists \alpha \in I : x \in A_\alpha\}$$

$$\bigcap_{\alpha \in I} A_\alpha = \{x \mid \forall \alpha \in I : x \in A_\alpha\}$$

$$\bigcup_{\alpha \in I} A_\alpha = \{x \mid \exists \alpha \in I : x \in A_\alpha\}$$

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$Q:\ I \neq \emptyset$ for $\bigcap_{\alpha \in I} A_\alpha$ (UD P_{91})

$$\bigcup_{\alpha \in I} A_\alpha = \{x \mid \exists \alpha \in I : x \in A_\alpha\}$$

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$Q:\ I \neq \emptyset$ for $\bigcup_{\alpha \in I} A_\alpha$

“$\bigcap_{n=1}^{\infty}$”: UD 8.1

$$A_n = [0, 1/n) \quad B_n = [0, 1/n] \quad C_n = (0, 1/n)$$

(b) Find $\bigcap_{n=1}^{\infty} A_n$ $\qquad \bigcap_{n=1}^{\infty} B_n$ $\qquad \bigcap_{n=1}^{\infty} C_n$

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Theorem (The Nested Interval Theorem (Cantor))

设 $\{[a_n, b_n]\}$ 为递降闭区间套序列, 即

$$[a_1, b_1] \supset [a_2, b_2] \supset \cdots \supset [a_n, b_n] \supset \cdots .$$

如果 $\lim_{n\to\infty}(b_n - a_n) = 0$, 则存在唯一的点 c, 使得 $c \in [a_n, b_n], \forall n \geq 1$.

"$\bigcap_{n=1}^{\infty}$": UD 8.4

$$\forall n \in \mathbb{Z}^+ : A_n \subset B_n \not\Rightarrow \bigcap_{n=1}^{\infty} A_n \subset \bigcap_{n=1}^{\infty} B_n$$

$$A_n = [0, 1/n) \quad B_n = [0, 1/n]$$

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DeMorgan's Law: UD Exercise 8.9

$$X \setminus \bigcup_{\alpha \in I} A_\alpha = \bigcap_{\alpha \in I} (X \setminus A_\alpha)$$

$$X \setminus \bigcap_{\alpha \in I} A_\alpha = \bigcup_{\alpha \in I} (X \setminus A_\alpha)$$

DeMorgan's Law: UD 8.8

$$A = \mathbb{R} \setminus \bigcap_{n \in \mathbb{Z}^+} (\mathbb{R} \setminus \{-n, -n+1, \cdots, 0, \cdots, n-1, n\})$$

DeMorgan's Law: UD 8.8

$$A = \mathbb{R} \setminus \bigcap_{n \in \mathbb{Z}^+} (\mathbb{R} \setminus \{-n, -n+1, \cdots, 0, \cdots, n-1, n\})$$

$$X_n = \{-n, -n+1, \cdots, 0, \cdots, n-1, n\}$$

DeMorgan's Law: UD 8.8

$$A = \mathbb{R} \setminus \bigcap_{n \in \mathbb{Z}^+} (\mathbb{R} \setminus \{-n, -n+1, \cdots, 0, \cdots, n-1, n\})$$

$$X_n = \{-n, -n+1, \cdots, 0, \cdots, n-1, n\}$$

$$\begin{aligned} A &= \mathbb{R} \setminus \bigcap_{n \in \mathbb{Z}^+} (\mathbb{R} \setminus X_n) \\ &= \mathbb{R} \setminus \Big(\mathbb{R} \setminus \bigcup_{n \in \mathbb{Z}^+} X_n\Big) \end{aligned}$$

DeMorgan's Law: UD 8.9

$$A = \mathbb{Q} \setminus \bigcap_{n \in \mathbb{Z}} (\mathbb{R} \setminus \{2n\})$$

DeMorgan's Law: UD 8.9

$$A = \mathbb{Q} \setminus \bigcap_{n \in \mathbb{Z}} (\mathbb{R} \setminus \{2n\})$$

Q : What is the temporary universe?

DeMorgan's Law: UD 8.9

$$A = \mathbb{Q} \setminus \bigcap_{n \in \mathbb{Z}} (\mathbb{R} \setminus \{2n\})$$

Q : What is the temporary universe?

$$\begin{aligned} A &= \mathbb{Q} \setminus \bigcap_{n \in \mathbb{Z}} (\mathbb{R} \setminus \{2n\}) \\ &= \mathbb{Q} \setminus \Big(\mathbb{R} \setminus \bigcup_{n \in \mathbb{Z}} \{2n\}\Big) \end{aligned}$$

Video:

Message To Future Generations — Bertrand Russell

Power Set

$$\{a,b,c\}$$

$$\left\{ \begin{array}{l} \{\}, \\ \{a\}, \{b\}, \{c\}, \\ \{a,b\}, \{a,c\}, \{b,c\}, \\ \{a,b,c\} \end{array} \right\}$$

Definition (Axiom of Power Set)

$$\forall X \exists Y \forall u (u \subseteq X \iff u \in Y)$$

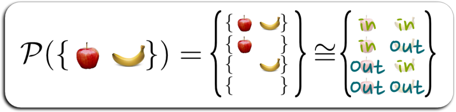
$$\mathcal{P}(X)$$

Definition (Axiom of Power Set)

$$\forall X \exists Y \forall u (u \subseteq X \iff u \in Y)$$

$$\mathcal{P}(X)$$

$$2^X = \{0, 1\}^X$$



$$\boxed{S \in \mathcal{P}(X) \iff S \subseteq X}$$

“$\subseteq$” (UD 9.4)

$$A \subseteq B \iff \mathcal{P}(A) \subseteq \mathcal{P}(B)$$

"⊆" (UD 9.4)

$$A \subseteq B \iff \mathcal{P}(A) \subseteq \mathcal{P}(B)$$

$$A \subseteq B \implies \mathcal{P}(A) \subseteq \mathcal{P}(B)$$

$$x \in \mathcal{P}(A) \implies x \subseteq A$$

$$\mathcal{P}(A) \subseteq \mathcal{P}(B) \implies A \subseteq B$$

$$x \in A \implies \{x\} \subseteq \mathcal{P}(A)$$

"$\subseteq$" (UD 9.2)

$$\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$$

“$\subseteq$” (UD 9.2)

$$\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$$

Proof.

$$A \subseteq B \iff \mathcal{P}(A) \subseteq \mathcal{P}(B)$$

“$\subseteq$” (UD 9.2)

$$\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$$

Proof.

$$A \subseteq B \iff \mathcal{P}(A) \subseteq \mathcal{P}(B)$$

$$\forall x \Big(x \in \mathcal{P}(A) \cup \mathcal{P}(B) \implies x \in \mathcal{P}(A \cup B) \Big)$$

$\square$

“⊆” (UD 9.2)

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□

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□

$$\mathcal{P}(A) \cup \mathcal{P}(B) \neq \mathcal{P}(A \cup B)$$

UD Exercise 9.3

$$\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$$


Thank
You!